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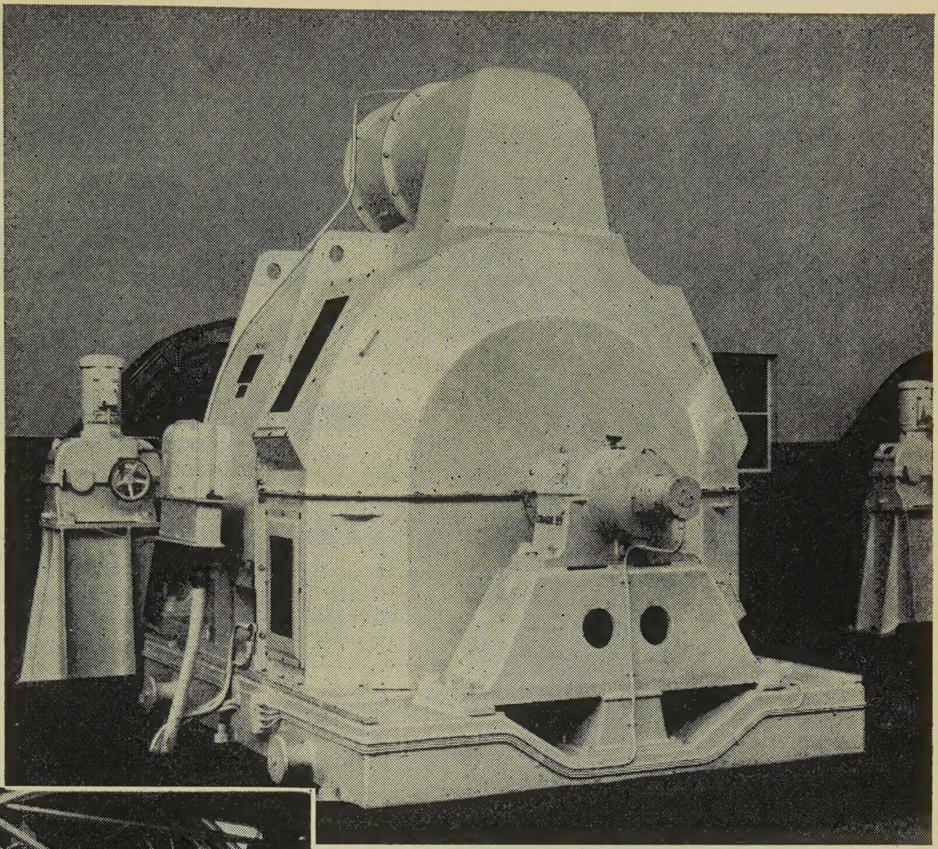

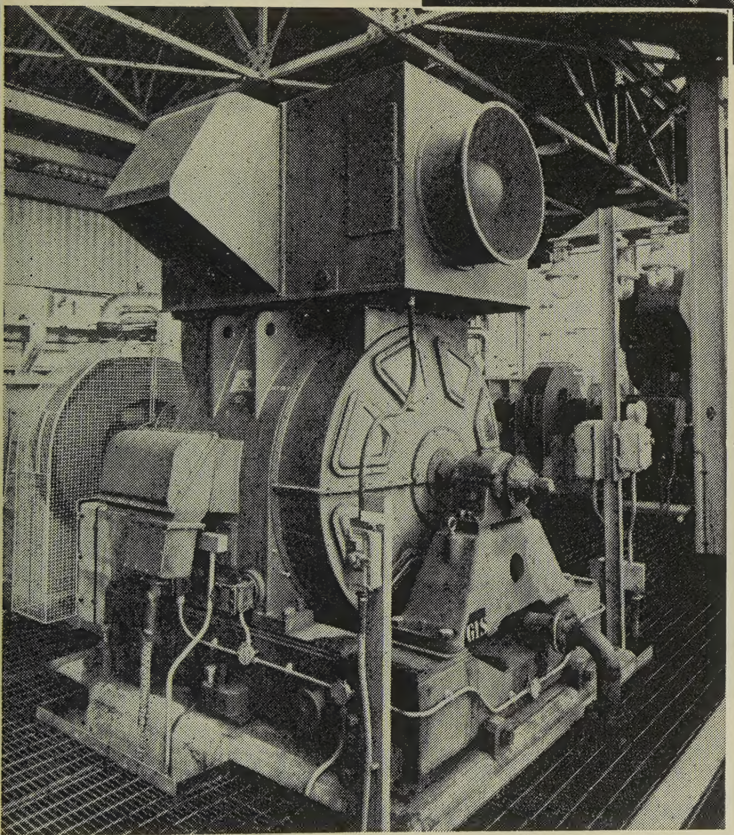

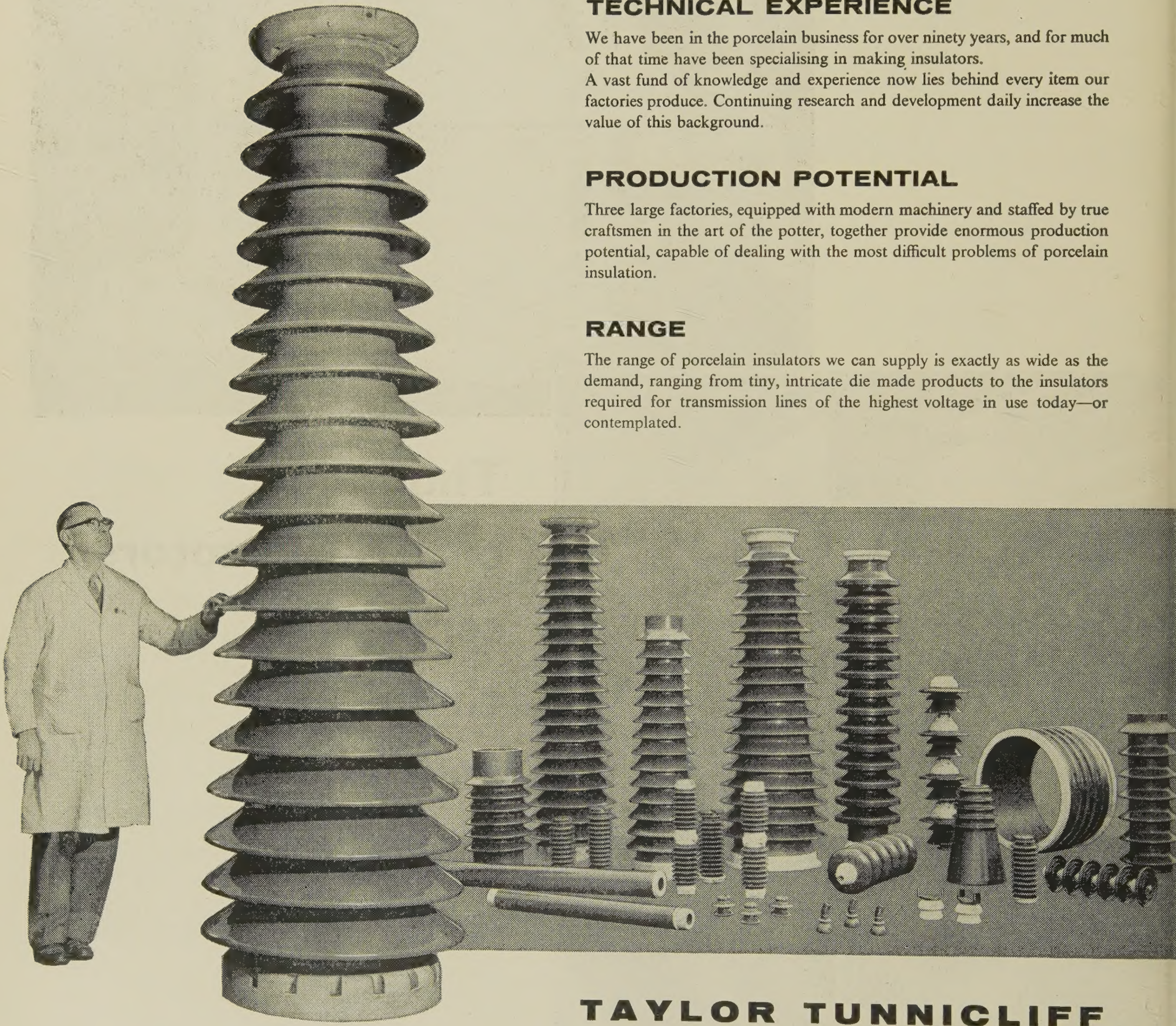

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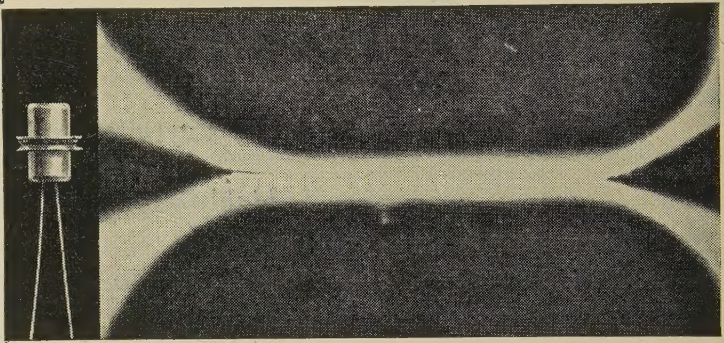

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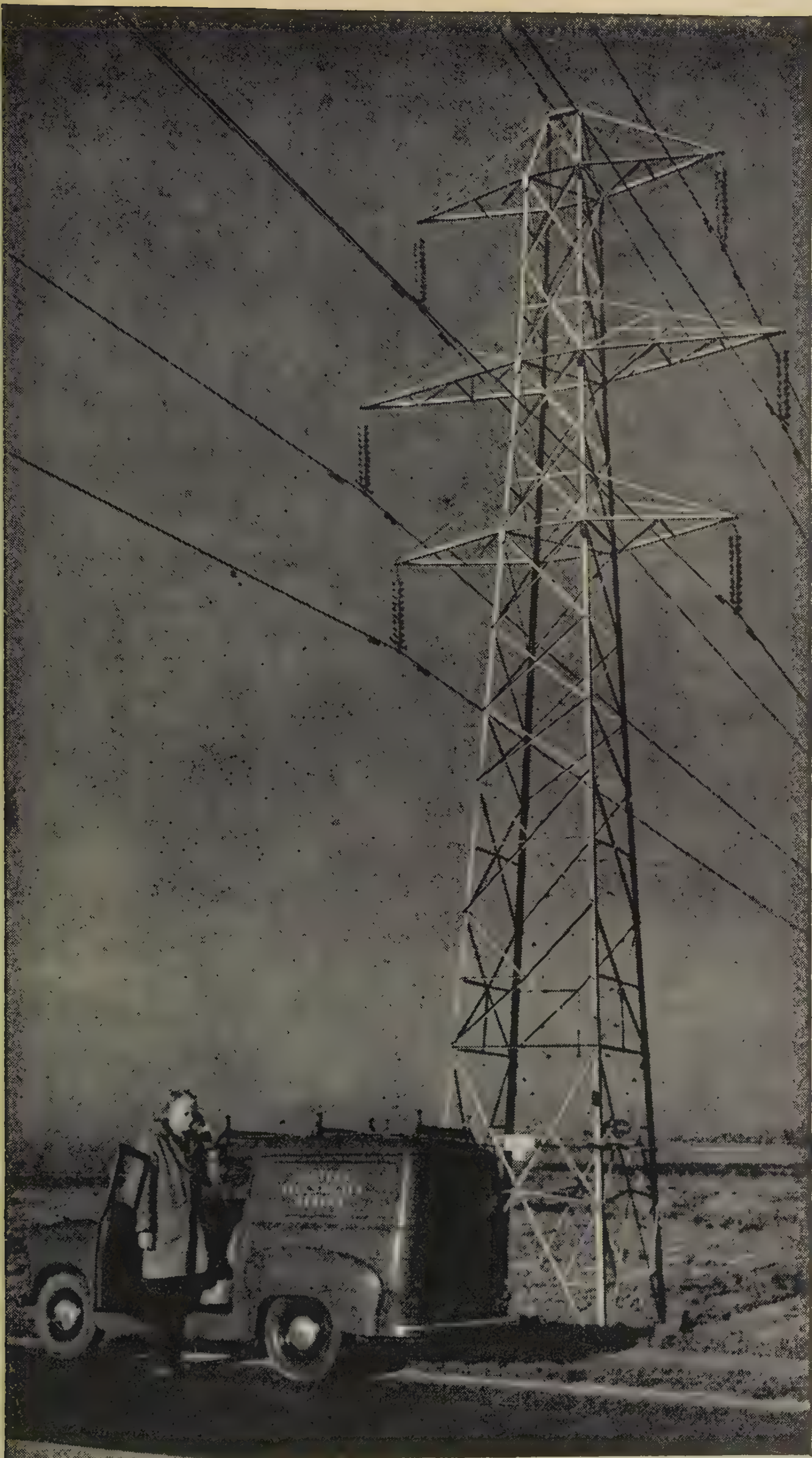

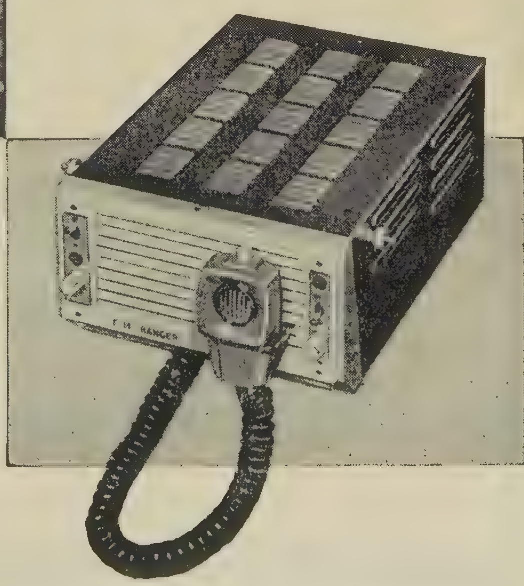

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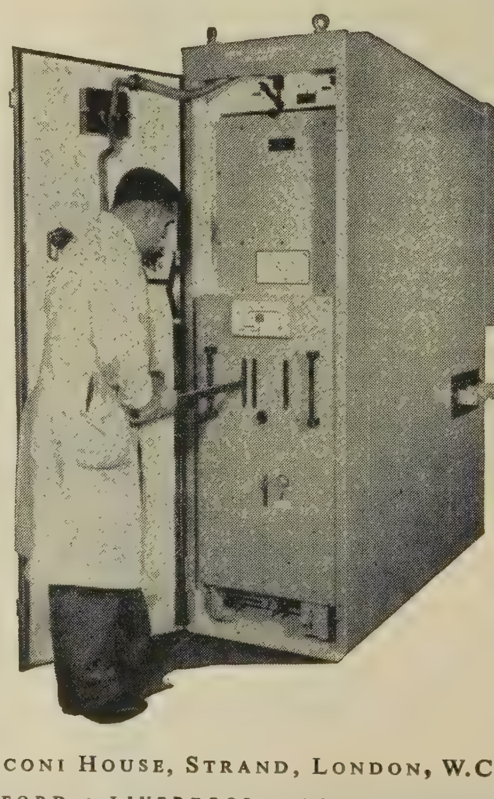

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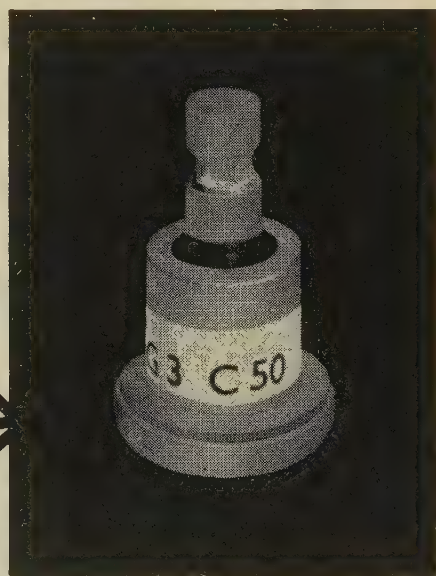

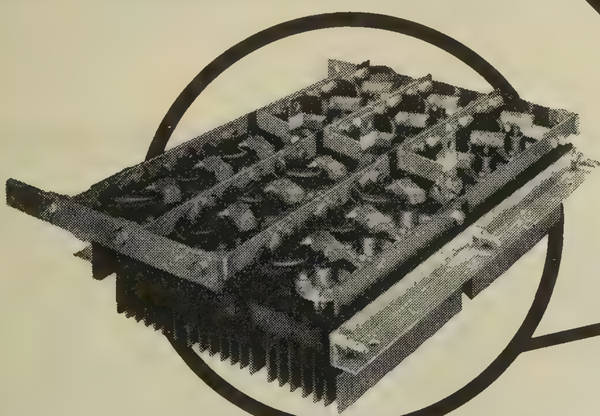

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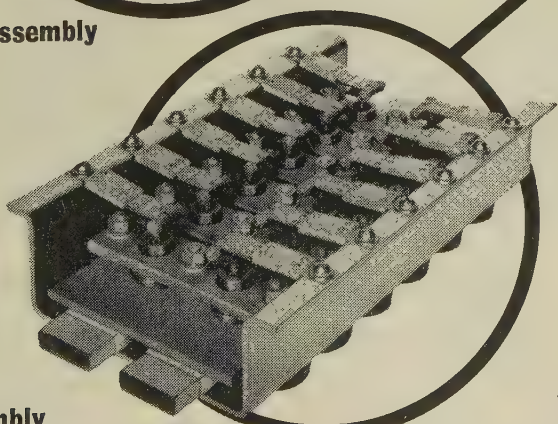

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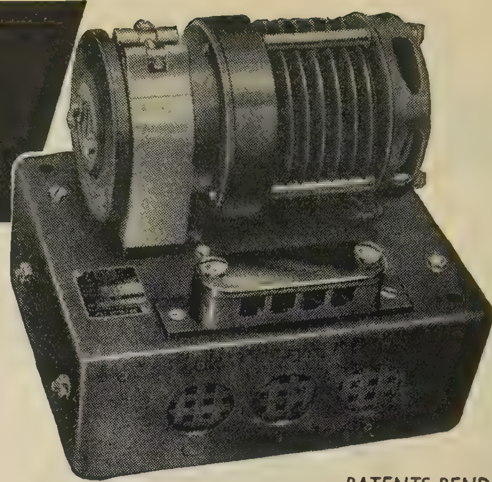

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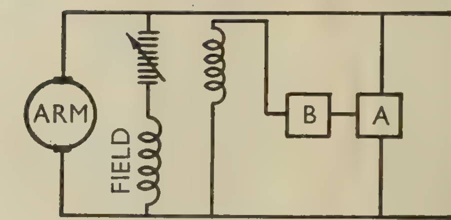

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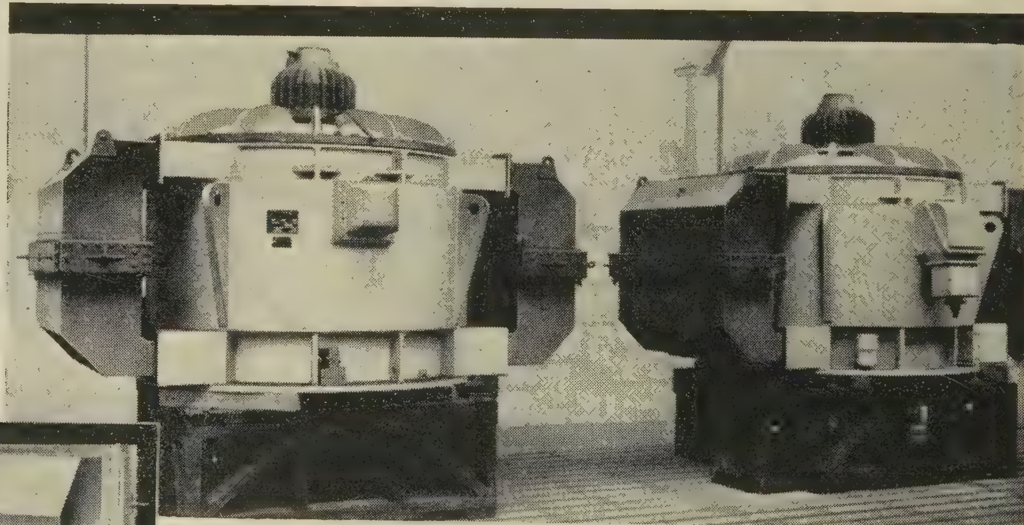

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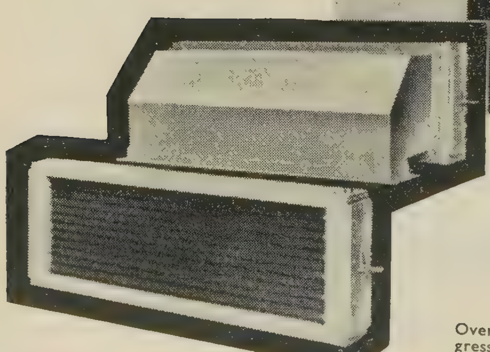

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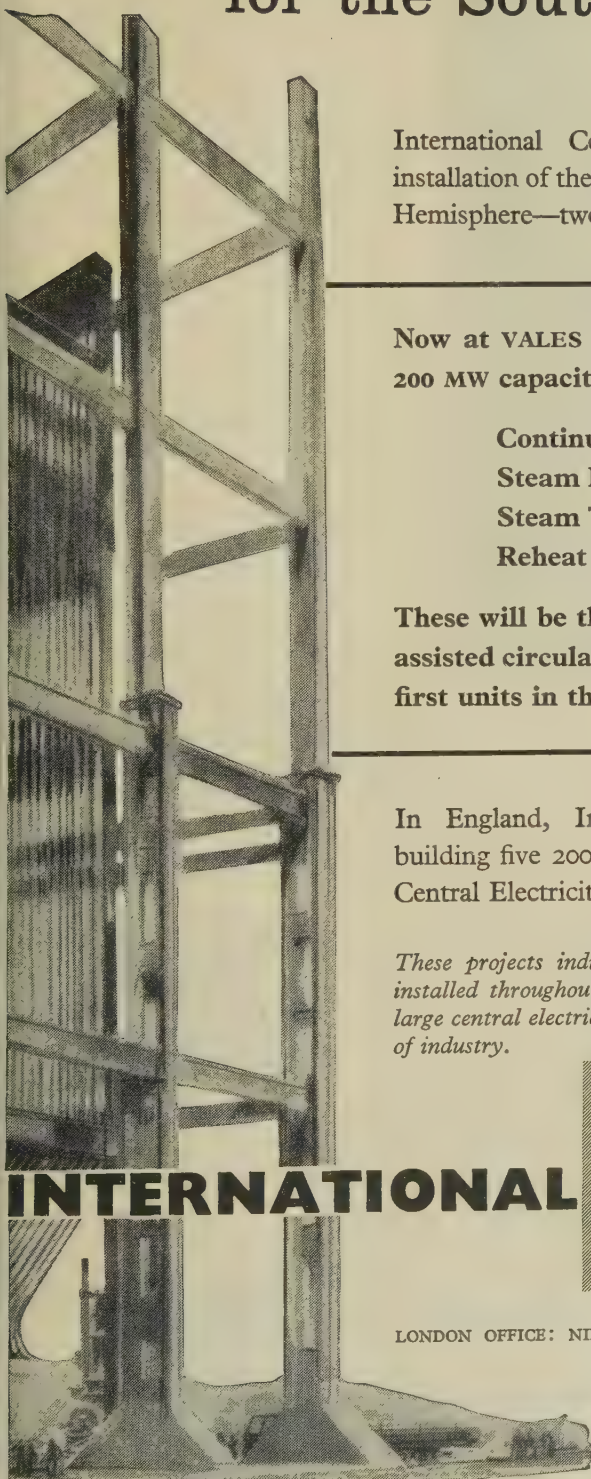

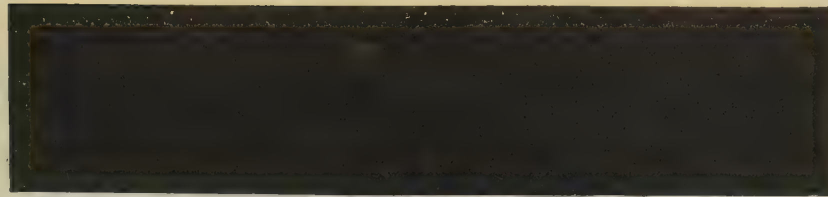

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Jan. 1960
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GASEOUS DISCHARGE PHENOMENA IN HIGH-VOLTAGE D.C. CABLE DIELECTRICS

By E. C. ROGERS, M.Sc., A.Inst.P., and D. J. SKIPPER, B.Sc.(Eng.), Associate Members.

(*The paper was first received* 9*th October*, 1958, *in revised form* 19*th February, and in final form* 18*th August*, 1959. *It was published in January*, 1960, *and was read before the* SUPPLY SECTION 13*th January*, 1960.)

SUMMARY

Considerations of the short-time electric strength and of the conductivity/temperature/stress relationships show that both impregnated paper and polythene are likely to be satisfactory dielectrics for use in h.v. d.c. cables. From a.c. experience, however, it is known that polythene in particular is very vulnerable to damage caused by discharges in gas-filled cavities, and an assessment of the importance of this mode of deterioration under d.c. conditions is therefore necessary.

Calculations have been made of the repetition rate of discharges in a gas-filled cavity in a dielectric subjected to a d.c. stress, since this is clearly a decisive factor determining the rate of deterioration. The repetition rate is shown to be a maximum when the surface conductivity of the dielectric is zero. Direct measurements of the repetition rate of discharges are described, and the measured rates are shown to be consistent with the calculated maximum rates. The effects of ripple voltages and polarity reversals are considered.

From the predicted discharge repetition rates and from the results of accelerated life tests on samples containing cavities, a rough estimate of the effect of discharges on the probable life of an h.v. d.c. cable dielectric can be made. Polythene was selected as the example for study, on account of its known vulnerability to discharges, and it is shown that, with this material, discharge damage is unlikely to be a serious problem under d.c. conditions, provided that ripple voltages are not excessive and that very frequent reversals of polarity can be avoided.

LIST OF PRINCIPAL SYMBOLS

z, r, ϕ = Cylindrical polar co-ordinates.
ξ, ζ, ϕ = Oblate spheroidal co-ordinates.
$P_n(\mu)$, $Q_n(\mu)$ = Legendre functions of the first and second kind.
V_1, V_2 = Potential functions inside and outside the cavity.
E' = Uniform stress applied to the dielectric.
E_1, E_2 = Stresses inside and outside the cavity.
E_ξ, E_ζ = Components of stress in oblate spheroidal co-ordinates.
E_r, E_z = Components of stress in cylindrical polar co-ordinates.
E_i = Discharge inception stress of the gas in the cavity.
J = Total current density, i.e. the conduction current plus the displacement current.
J_ξ, J_ζ = Components of total current density in oblate spheroidal co-ordinates.
ϵ_1, ϵ_2 = Relative permittivities of the media inside and outside the cavity.
ϵ_0 = Permittivity of free space.
σ_1, σ_2 = Conductivities of the media inside and outside the cavity.
σ_s = Surface conductivity of the cavity boundary.
a, b = Semi-axis and radius of the oblate spheroidal cavity.
c = Radius of the focal circle of the cavity.
T = Interval between successive discharges.
T' = Minimum interval between successive discharges.
t = Time.

A notation of the form X (x, y, z) has been used to indicate that X is a function of the variables x, y, z, and X (a, y, z), where a is a constant, denotes that values of X on the surface $x = a$ are being considered.

The rationalized M.K.S. system of units has been used.

(1) INTRODUCTION

Interest in h.v. d.c. cables has been stimulated in recent years by a number of projects involving the interconnection of large power systems by underwater routes. With a.c. transmission, the permissible length of a cable connection is limited by the charging current, and the maximum distances have been estimated[1] at approximately 40 miles for 132kV, 25 miles for 220kV and 15 miles for 400kV. With land cables, reactive compensation may be provided at intermediate points, but this is impracticable with submarine installations, and for submarine connections over distances much greater than these critical lengths d.c. transmission must be used. The 100kV d.c. submarine cable linking the island of Gotland with the Swedish mainland[2] 60 miles distant is an example of such a connection.

Mr. Rogers is, and Mr. Skipper was formerly, with British Insulated Callender's Cables Ltd.
Mr. Skipper is now with the Central Electricity Generating Board.

Even when the distance is less than the limiting value, there may be other reasons for selecting d.c. transmission, as in the proposed scheme to link the British and French electricity systems by a cable across the Channel, transmitting about 160 MW at ± 100kV to earth. There the distance does not preclude the use of alternating currents, and the decision to use direct currents was based partly on the difficulty of synchronizing the frequencies of the two systems. On land, the considerably greater cost of cables makes it improbable that they will be used where conditions permit the use of overhead lines, and it seems likely that, in the immediate future, the major application of h.v. d.c. cables will be for high power-transfer-capacity interconnections between large power systems by underwater routes.

One of the factors that must be considered in the design of h.v. d.c. cables is the possible effect of discharges in gas-filled cavities on the life of the dielectric. The importance of this effect under a.c. conditions has long been recognized, and has resulted in the evolution of the modern pressure-assisted h.v. a.c. cable, in which discharges in the impregnated-paper dielectric are suppressed by the application of gas or oil pressure. The difficulty of eliminating discharges has also, up to the present, prevented the use of thermoplastic insulants, such as polythene, in a.c. cables for operation at voltages much greater than 33 kV.

The rate at which a dielectric is damaged by discharges in a gas-filled cavity is determined by the repetition rate of the discharges. At 50 c/s, when the discharge-inception stress is applied, the repetition rate has a minimum value of 100 discharges per site per second, since at least one discharge occurs in each half-cycle, and as the stress is raised above this level the rate increases.[3] When a d.c. stress is applied the discharge sequence is quite different. There is first a rapid succession of discharges as the stress is raised from zero to the steady value, and subsequently, with a perfect dielectric having infinite resistivity, no further discharges will occur provided that the stress remains steady. In practice all dielectrics have a finite resistivity, and so, even when the stress is steady, repetitive discharges do occur, at intervals determined by the time required for the cavity to be recharged to the breakdown voltage by conduction through the dielectric.

Although there is little doubt that, under normal operating conditions, the repetition rate of discharges in cavities in d.c. cable dielectrics will be very much less than the minimum 50 c/s repetition rate, it is by no means certain that their damaging effect will be negligible. Information on the subject in the literature is scanty, and the work described in the paper was therefore undertaken with the object of establishing the repetition rate of discharges under d.c. conditions and so permitting an assessment of the importance of discharge damage as a mode of deterioration of h.v. d.c. cable dielectrics.

To place this work in proper perspective, a brief account is first given of the electrical requirements for the dielectric of an h.v. d.c. cable, in the light of which the suitability of various materials is discussed.

(2) DIELECTRICS FOR H.V. D.C. CABLES

(2.1) Electrical Requirements

Since the permittivity of most cable dielectrics is virtually independent of temperature and electric stress, the stress distribution in an a.c. cable is determined almost entirely by the cable geometry and does not vary significantly with load. In a d.c. cable, however, the steady-state stress distribution is markedly dependent on load, since it is determined by the conductivity of the dielectric, which, in general, varies with both temperature and stress. This dependence of conductivity on both temperature and stress is illustrated, for paper pre-

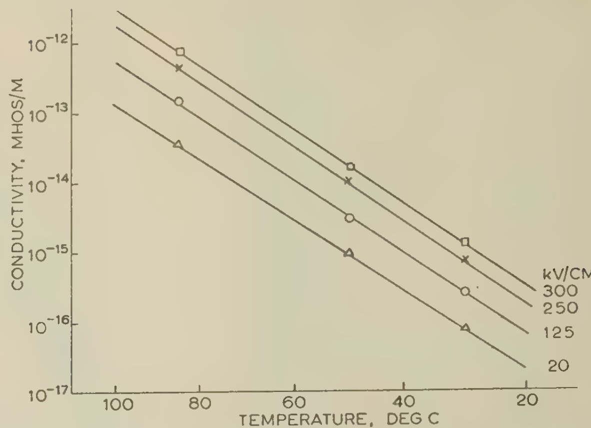


Fig. 1.—Conductivity/temperature curves for paper pre-impregnated with petroleum jelly.

Each point represents the mean of at least 9 individual results.

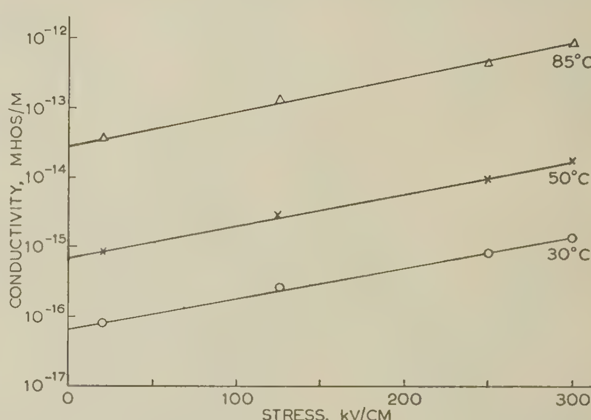


Fig. 2.—Conductivity/stress curves for paper pre-impregnated with petroleum jelly.

Each point represents the mean of at least 9 individual results.

impregnated with petroleum jelly, in Figs. 1 and 2, respectively. Since the conductivity scales in the two Figures are logarithmic, the parallelism of the lines representing different stress and temperature levels implies that there is no appreciable interaction between the effects of stress and temperature on conductivity. This has been found to be true for most of the dielectrics studied, and simplifies the computation of the stress distribution in d.c. cable dielectrics.

If the conductivity of the dielectric were constant, the stress distribution in a d.c. cable would be hyperbolic, as with alternating current. However, under steady-state conditions the temperature dependence of the conductivity has the effect of reducing the stress at the conductor, where the temperature is highest, and increasing it at the sheath. It is thus possible, in a d.c. cable, for the stress at the sheath to exceed considerably that at the conductor, an effect which is termed 'stress inversion'. The effect is alleviated to some extent by any stress dependence of the conductivity, which results in greater uniformity of the stress distribution under all load conditions. With a cable on load, a certain measure of stress inversion may be advantageous, since the maximum stress is transferred to the cooler regions near the sheath, which are intrinsically stronger. However, too high a sheath stress may lead to difficulty in the design of joints.

In general, the requirements for a satisfactory stress distribution are a low thermal resistivity (which is also desirable from considerations of current rating) and an electric conductivity

which is not too dependent on temperature over the likely working range. Other electrical characteristics which must be considered in assessing the suitability of a dielectric for use in h.v. d.c. cables are the impulse and d.c. breakdown strengths and possible mechanisms of long-term deterioration. Among these are electrochemical action and deterioration caused by discharges in gas-filled cavities. With rubber and plastic dielectrics (for which, in submarine applications, the omission of the metal sheath would have obvious advantages) the effect of water absorption on the electrical characteristics must also be considered.

(2.2) Assessment of Various Dielectrics

In terms of the electrical requirements outlined above, the suitability of various dielectrics for use in h.v. d.c. cables will now be briefly considered.

(2.2.1) Polyvinyl Chloride.

The conductivity of polyvinyl chloride (p.v.c.) is greatly influenced by the type and amount of plasticizer incorporated in the compound, but is markedly dependent on temperature, the conductivity of a typical compound investigated by the authors being 10^{-13} mho/m at 20° C and 5×10^{-10} mho/m at 85° C. This marked dependence of conductivity on temperature was not accompanied by any significant variation with stress, and the resulting stress inversion on load would be further accentuated by the comparatively high thermal resistivity (about 8 thermal ohm-m) which would also adversely affect the current rating. Furthermore, immersion in water caused a marked increase in conductivity, which, in a d.c. cable, could lead to failure by electrochemical action at comparatively low stresses.[10] Hence, p.v.c. is unlikely to be a suitable dielectric for use in h.v. d.c. cables.

(2.2.2) Butyl Rubber.

The authors' measurements show that butyl rubber has conductivity/temperature/stress characteristics which, together with a low thermal resistivity (about 4 thermal ohm-m) would result in a satisfactory stress distribution on load. However, the d.c. strength (measured on flat sheets at 80°C) is only about 400 kV/cm, or little more than one-third of that of impregnated paper, and it is further reduced by prolonged immersion in water.

(2.2.3) Terylene and Polystyrene.

Both Terylene and polystyrene are electrically satisfactory, but, for the present, they must be ruled out because their cost is relatively high.

(2.2.4) Impregnated Paper.

At the present time, impregnated paper is most favoured as the dielectric for h.v. d.c. cables, not only because of its satisfactory electrical properties, but also because of the vast amount of operating experience which has been gained from a.c. installations. All forms of impregnated paper have conductivity/temperature/stress characteristics which lead to satisfactory stress distributions in cables on load, and the small amount of operating experience obtained to date with direct voltages[2,4] shows that solid-type oil/paper dielectric behaves quite satisfactorily at conductor stresses between 150 and 220 kV/cm. (These conductor stresses have been calculated assuming a hyperbolic distribution, merely for comparison with a.c. cables.)

Service experience of the relative merits of solid-type and pressure-assisted cables for d.c. use is completely lacking, although, as discharges are likely to be a very much less serious problem under d.c. conditions than with a.c., the advantages of pressurization would be expected to be correspondingly less. Some French tests[4] gave asymptotic breakdown strengths of about 900 kV/cm for both solid and oil-filled cables. However, in some Swedish tests,[5] in which cables were subjected to loading cycles in which the applied voltage was increased in steps, solid-type cables failed at stresses of 300–400kV/cm, whereas comparable oil-filled cables withstood stresses of 500–600kV/cm for a considerable number of loading cycles. The solid-type cables were tested in the form of an inverted U, which encouraged compound migration from the crest of the U, and the failures occurred at these points. However, these two sets of results are not necessarily inconsistent, since they illustrate that, although liability to damage by discharges is much less with direct current than with alternating current, nevertheless, under conditions favourable to void formation, discharge damage is an important factor limiting the d.c. life of impregnated paper dielectric.

(2.2.5) Polythene.

The low thermal resistivity of polythene of about 3·5 thermal ohm-m is favourable to current rating, and, together with the satisfactory conductivity/temperature/stress characteristics, would result in an almost uniform stress distribution on load. Moreover, the electrical properties are not seriously affected by prolonged immersion in water.[6] Polythene is therefore a very attractive material for use as an insulant for h.v. d.c. submarine cables, since a water-resistant sheath would not be needed. The conductivity decreases slightly with increasing molecular weight, and a high-molecular-weight grade of polythene, such as Alkathene grade 0·3, is preferred, on account of its lower conductivity and also because of its inherent resistance to environmental stress cracking.

The permissible working stress in polythene cable will probably be limited by the relatively low impulse and short-time d.c. breakdown strengths, and, although strengths of the order of 7MV/cm have been recorded in tests on thin laboratory samples of polythene,[7] the range of impulse breakdown stresses obtained on thick-walled cables at 20°C is about 700–1 000kv/cm.[8, 9, 10] It has also been reported[9] that the d.c. breakdown stress of a polythene cable (expressed as a mean stress) is 500–600 kV/cm and that this is decreased by 50% as the temperature is raised from 20 to 80° C.

Polythene is extremely vulnerable to discharge damage, and this vulnerability, together with the fact that it has not so far been possible to devise a cable construction in which complete freedom from cavities can be guaranteed, has prevented its use in a.c. cables for operation at voltages much greater than 33 kV. The d.c. strength should be adequate to permit polythene-insulated h.v. d.c. cables to be operated with mean stresses in the dielectric in the region of 150 kV/cm,[11] but in view of the susceptibility to discharge damage, an assessment of the probable effects of cavities on the d.c. life is obviously essential.

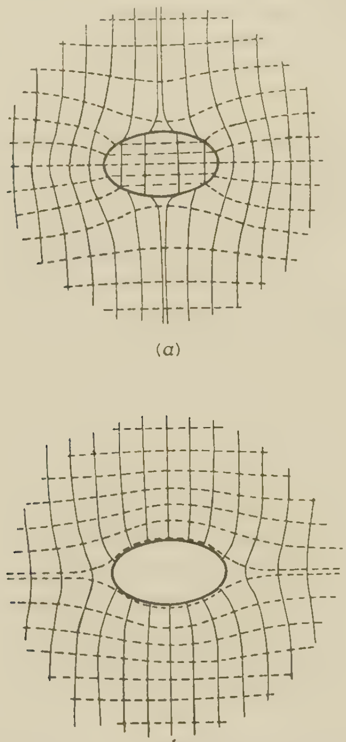
In the Sections that follow, the repetition rate of discharges in a cavity in a dielectric subjected to a d.c. stress is first derived theoretically, and then a description is given of the experimental determination of the repetition rate of discharges in cavities in polythene. This material was selected in preference to impregnated paper on account of its greater susceptibility to discharge damage, and also on account of its translucence, which permitted the use of an optical method of discharge detection.

(3) THEORETICAL ESTIMATE OF THE DISCHARGE REPETITION RATE

(3.1) Theoretical Model

The theoretical model considered is that of a uniformly stressed infinite dielectric medium containing a single oblate spheroidal cavity with its axis of symmetry parallel to the field direction. For simplicity, the volume and surface conductivities

of the dielectric are assumed to be ohmic. In the absence of the cavity, the field in the dielectric would be uniform and the current density the same at all points. The cavity disturbs the current flow, however, so that an excess of charge develops on one cavity face and a deficiency on the other. These surface charges, which consist partly of free charge and partly of bound charge produced by polarization of the dielectric, distort the electric field in such a way as to cause the current to flow around the cavity instead of accumulating on its faces, so that the field tends towards the steady-state condition shown in Fig. 3(*a*).



(*a*)

(*b*)

Fig. 3.—Field distribution for a single oblate-spheroidal cavity in a uniform infinite dielectric medium.

(*a*) Steady-state field.
(*b*) Field after complete discharge of the cavity.
——— Lines of force.
- - - - Equipotential surfaces.

Up to this point it has been assumed that the gas in the cavity is non-conducting. If, however, the steady-state stress in the cavity exceeds the discharge inception value, the gas will break down when this critical value of stress is reached. For simplicity, suppose that the cavity discharges completely and uniformly, so that in a very short interval of time (according to Whitehead,[12] less than 10^{-7} sec) the stress at all points in the cavity is reduced to zero. The field distribution is then as shown in Fig. 3(*b*). When, on completion of the discharge, the gas ceases to conduct, the field distribution once again tends towards the steady-state condition shown in Fig. 3(*a*), until the inception stress is reached, another discharge occurs and the process is repeated. The repetition rate of the discharges is therefore determined by the time required for the stress in the cavity to build up from zero to the discharge inception value.

The applied field is perturbed solely by surface charges on the cavity boundary, and at no time are there distributions of volume charge either in the region outside the cavity or, except at the instant of discharge, within the cavity. The potential functions in the two regions therefore both satisfy Laplace's equation, and the transient field distribution may be derived by expressing the two potential functions in harmonic form, and imposing a time-dependent boundary condition.

(3.2) Co-ordinate System

Referring to Fig. 4, the equation of the cavity surface in cylindrical polar co-ordinates (z, r, ϕ) is

$$(z/a)^2 + (r/b)^2 = 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

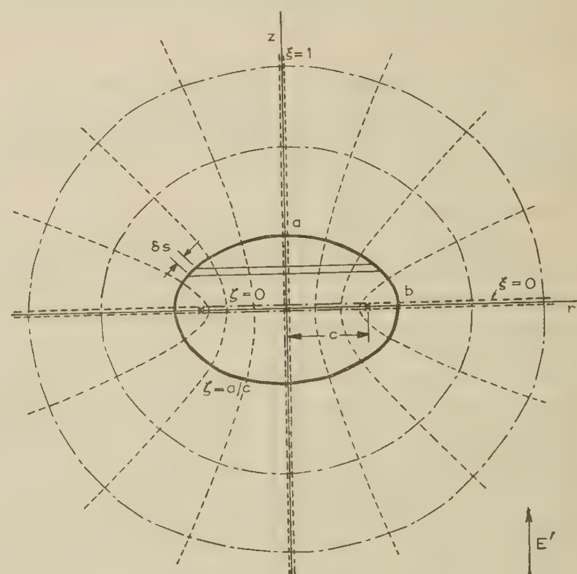


Fig. 4.—Co-ordinate surfaces.

- - - - - Confocal hyperboloids.
—— —— Confocal oblate spheroids.

Instead of the cylindrical polar co-ordinates (z, r, ϕ) it is preferable to use oblate spheroidal co-ordinates (ξ, ζ, ϕ), since this simplifies the imposition of the boundary conditions. These co-ordinates will now be defined.

Consider the set of surfaces given by

$$\frac{z^2}{c^2\zeta^2} + \frac{r^2}{c^2(\zeta^2 + 1)} = 1 \quad . \quad . \quad . \quad . \quad (2)$$

where $\zeta > 0$.

As the parameter ζ tends to zero, the surface collapses to a disc normal to the z-axis, of radius c and with centre at the origin (see Fig. 4), and as ζ tends to infinity, the surface tends to a sphere of infinite radius. For intermediate values of ζ, the surfaces consist of confocal oblate spheroids, and a single unique spheroid passes through each point in space. The parameter ζ is the length of the semi-axis of the spheroidal surface divided by c, the radius of the focal circle. Hence, if c is chosen so that the cavity surface is a member of the set defined by eqn. (2), the equation of the surface can be expressed in the simple form $\zeta = a/c$.

Now consider the set of surfaces given by

$$\frac{-z^2}{c^2\xi^2} + \frac{r^2}{c^2(1 - \xi^2)} = 1 \quad . \quad . \quad . \quad . \quad (3)$$

where $0 < \xi < 1$.

As the parameter ξ tends to zero, the surface collapses on to a plane through the origin, normal to the z-axis, and consists of the outer region of the circle of radius c obtained by putting $\zeta = 0$ in eqn. (2). As ξ tends to unity, the surface collapses to a line on the z-axis, or axis of symmetry. For intermediate values of ξ, the surfaces consist of confocal single-sheet hyper-

boloids, and a single unique hyperboloid passes through each point in space. The parameter ξ is the cosine of the semi-angle of the cone asymptotic to the hyperboloidal surface.

The third oblate spheroidal co-ordinate, ϕ, is the angle of longitude, and is defined in the same way as for cylindrical polar co-ordinates. The problem under consideration is axially symmetric, however, so that this co-ordinate is not required.

It may be shown that the set of confocal spheroids, given by eqn. (2), and the set of confocal hyperboloids, given by eqn. (3), are mutually orthogonal surfaces, and if (ξ, ζ), $(\xi + \delta\xi. \zeta + \delta\zeta)$ are the co-ordinates of two points lying in a plane through the axis of symmetry, then δs, the distance between them, is given by

$$\delta s^2 = h_1^2\delta\xi^2 + h_2^2\delta\zeta^2 \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where

$$h_1^2 = c^2(\xi^2 + \zeta^2)/(1 - \xi^2) \quad . \quad . \quad . \quad . \quad (5)$$

$$h_2^2 = c^2(\xi^2 + \zeta^2)/(1 + \zeta^2) \quad . \quad . \quad . \quad . \quad (6)$$

Also, if (z, r), (ξ, ζ) are respectively the cylindrical polar and oblate spheroidal co-ordinates of a point, then

$$z = c\xi\zeta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$r = c[(1 + \zeta^2)(1 - \xi^2)]^{1/2} \quad . \quad . \quad . \quad (8)$$

(3.3) Potential Functions

The general solution of Laplace's equation in oblate spheroidal co-ordinates (ξ, ζ) may be shown to be[13]

$$V = \sum_n [A_n\mathrm{P}_n(j\zeta) + B_n\mathrm{Q}_n(j\zeta)][C_n\mathrm{P}_n(\xi) + D_n\mathrm{Q}_n(\xi)] \qquad (9)$$

where A_n, B_n, C_n, D_n are arbitrary constants.

The potential functions inside and outside the cavity, V_1 and V_2, may be expressed in the form of harmonic series of this type, except that the coefficients A_n, B_n, . . . will not be constants, but functions of time. Suitable harmonic terms must be selected, subject to the following boundary conditions:

(*a*) V_1 and V_2, and their space derivatives, must be finite and continuous at all points in their respective regions.

(*b*) $V_1 = V_2$ at all points on the cavity boundary.

(*c*) At points remote from the cavity, the field intensity E_2, must tend to the steady applied value E'. Therefore as ζ tends to infinity, V_2 must tend to $-E'z$, or $-E'c\xi\zeta$.

(*d*) In order to provide a reference level for the potentials, the plane equipotential surface through the equator of the cavity is defined to be at zero potential. Therefore V_1 and V_2 must be zero for all ξ when $\zeta = 0$, and for all ζ when $\xi = 0$.

The following expressions satisfy these conditions:

$$V_1 = -E'c\xi\zeta + \sum_{n\ odd} A_n[\mathrm{Q}_n(j\alpha)/\mathrm{P}_n(j\alpha)]\mathrm{P}_n(j\zeta)\mathrm{P}_n(\xi) \quad (10)$$

$$V_2 = -E'c\xi\zeta + \sum_{n\ odd} A_n\mathrm{Q}_n(j\zeta)\mathrm{P}_n(\xi) \quad . \quad . \quad . \quad . \quad (11)$$

where $\alpha = a/c$.

In each expression, the term $-E'c\ \xi\zeta$ represents the steady applied field, and the harmonic series represents the perturbing field produced by the distribution of surface charge on the cavity boundary.

(3.4) Initial Field

Let a discharge occur at time $t = 0$, so that the stress at all points in the cavity is reduced to zero and the cavity boundary becomes an equipotential surface at zero potential. The lines of force and equipotential surfaces are then as shown in Fig. 3(*b*), and V_1 must be zero for all ξ and ζ.

The values of A_n, in eqns. (10) and (11) which satisfy this condition are

$$A_1 = \frac{E'c\alpha}{\alpha \operatorname{arc\,cot} \alpha - 1} \quad . \quad . \quad . \quad . \quad (12)$$

$$A_n = 0 \quad n > 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

(3.5) Transient Field

(3.5.1) Time-Dependent Boundary Condition.

After the discharge, the field distribution tends towards the steady-state condition shown in Fig. 3(*a*). The potentials in the regions inside and outside the cavity may still be represented by the harmonic series given in eqns. (10) and (11), but the terms A_1, A_3, A_5, . . . will now be functions of time. The boundary condition which determines these functions will now be derived.

Referring to Fig. 4, consider an annular element of the cavity surface, of width δs, the oblate spheroidal co-ordinates of its edges being (ξ, α) and $(\xi + \delta\xi, \alpha)$. The divergence of the total current, J, over a closed annular surface fitting tightly to this element must be zero, so that

$$J_{\zeta 1}(\xi, \alpha, t) - J_{\zeta 2}(\xi, \alpha, t) = \frac{1}{h_1(\xi, \alpha)}\frac{\partial}{\partial\xi}J_s(\xi, t) \quad . \quad (14)$$

where

$$J_{\zeta 1}(\xi, \alpha, t) = -\frac{1}{h_2(\xi, \alpha)}\left(\sigma_1 + \epsilon_0\epsilon_1\frac{\partial}{\partial t}\right)\left(\frac{\partial V_1}{\partial\zeta}\right)_{\zeta=\alpha} \quad (15)$$

$$J_{\zeta 2}(\xi, \alpha, t) = -\frac{1}{h_2(\xi, \alpha)}\left(\sigma_2 + \epsilon_0\epsilon_2\frac{\partial}{\partial t}\right)\left(\frac{\partial V_2}{\partial\zeta}\right)_{\zeta=\alpha} \quad (16)$$

$$J_s(\xi, t) = -\frac{1}{h_1(\xi, \alpha)}\sigma_s(\xi)\left(\frac{\partial V_2}{\partial\xi}\right)_{\zeta=\alpha} \quad . \quad . \quad . \quad . \quad (17)$$

Eqn. (14) is the boundary condition which determines the functions $A_n(t)$.

(3.5.2) Surface Conductivity.

The form to be assumed for the surface conductivity $\sigma_s(\xi)$ must now be considered. It might at first appear preferable to take σ_s constant over the entire cavity surface. However, this results in non-terminating solutions for the potentials and a non-uniform stress distribution in the cavity. There is no reason to suppose that the surface conductivity would, in fact, be strictly uniform, and, for simplicity, it is preferable to take

$$\sigma_s = \sigma_{s0}[(\alpha^2 + \xi^2)/(1 + \alpha^2)]^{1/2} = \sigma_{s0}h_2(\xi, \alpha)/c$$

The conductivity is then σ_{s0} at the poles of the cavity and decreases to $\sigma_{s0}a/b$ at the equator.

(3.5.3) Derivation of the Functions $A_n(t)$.

The functions $A_n(t)$ are derived by substituting in eqn. (14) the expressions for V_1 and V_2 given by eqns. (10) and (11), and equating coefficients of $\mathrm{P}_n(\xi)$ to zero. The result obtained is that

$$A_n(t) = 0 \quad n > 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

$$A_1(t) = [\gamma + A_1(0)]\varepsilon^{-t/\tau} - \gamma \quad . \quad . \quad . \quad (19)$$

where

$$\gamma = \frac{-E'c\alpha[(\sigma_{s0}\alpha/c) + (\sigma_1 - \sigma_2)(1 + \alpha^2)]}{[(\sigma_{s0}\alpha/c) + (\sigma_1 - \sigma_2)(1 + \alpha^2)](\alpha \operatorname{arc\,cot} \alpha - 1) - \sigma_2} \quad (20)$$

$$\tau = \frac{\epsilon_0(\epsilon_1 - \epsilon_2)(1 + \alpha^2)(\alpha \operatorname{arc\,cot} \alpha - 1) - \epsilon_0\epsilon_2}{[(\sigma_{s0}\alpha/c) + (\sigma_1 - \sigma_2)(1 + \alpha^2)](\alpha \operatorname{arc\,cot} \alpha - 1) - \sigma_2} \quad (21)$$

and, as shown in Section 3.4, $A_1(0) = E'c\alpha/(\alpha \operatorname{arc\,cot} \alpha - 1)$.

(3.6) Stress in the Cavity

The stress in the cavity may now be determined as a function of time. Differentiation of eqn. (10) and substitution of the values of $A_n(t)$ given in Section 3.5.3 yields the result that

$$E_{r1} = -\frac{\partial V_1}{\partial r} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

$$E_{z1} = -\frac{\partial V_1}{\partial z} = E'$$
$$+ \{\gamma - [\gamma + A_1(0)]\varepsilon^{-t/\tau}\}\,(\alpha \operatorname{arc\,cot} \alpha - 1)/\alpha c \quad (23)$$

The stress in the cavity is therefore uniform and parallel to the z-axis at all points. When the discharge occurs, at time $t = 0$, the stress is zero and it then increases exponentially with time-constant, τ, tending towards the steady-state value given by

$$\lim_{t\to\infty} E_1 = E' + \gamma(\alpha \operatorname{arc\,cot} \alpha - 1)/\alpha c \quad . \quad . \quad (24)$$
$$= E'\lambda \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

where

$$1/\lambda = 1 - (1/\sigma_2)(\alpha \operatorname{arc\,cot} \alpha - 1)[(\sigma_{s0}\alpha/c) + (\sigma_1 - \sigma_2)(1 + \alpha^2)] \quad . \quad . \quad . \quad . \quad (26)$$

(3.7) Discharge Repetition Rate

The stress in the cavity may be expressed in the form $E_1(t) = E'\lambda(1 - \varepsilon^{-t/\tau})$, where λ and τ are given by eqns. (26) and (21). If T is the time required for the stress to rise from zero to the discharge inception value E_i, then

$$T = -\tau \log_\varepsilon\left(1 - \frac{E_i}{\lambda E'}\right) \quad . \quad . \quad . \quad (27)$$

The terms τ and λ are both functions of σ_{s0}, the surface conductivity of the cavity boundary, which will in general be unknown, since it is not an intrinsic property of the dielectric but depends on absorbed surface layers. It is shown in the Appendix, however, that the discharge repetition rate is a maximum when $\sigma_{s0} = 0$. A maximum possible repetition rate can therefore be specified in terms of the volume conductivity and permittivity of the dielectric, the cavity dimensions and the ratio of the stress in the dielectric to the discharge-inception stress of the gas in the cavity, all of which will, in general, be known quantities. T', the minimum value of T, can be obtained from eqn. (27) by putting $\sigma_{s0} = 0$ in the expressions for τ and λ. After putting $\sigma_1 = 0$, $\epsilon_1 = 1$ and $\epsilon_0 = 8\cdot85 \times 10^{-12}$ F/m, the final result is that

$$T' = 8\cdot85 \times 10^{-12}(1/\sigma_2)(1 - \epsilon_2 - \lambda')\log_\varepsilon\left(1 - \frac{E_i}{\lambda' E'}\right) \quad (28)$$

where $1/\lambda' = (\alpha \operatorname{arc\,cot} \alpha - 1)(1 + \alpha^2) + 1$

and $\alpha = a/c = [(b/a)^2 - 1]^{-1/2}$

In this expression, T' is in seconds and σ_2 is in mhos per metre.

In practice, the applied field, E', is likely to be considerably greater than the inception stress, E_i, and λ' is greater than $1\cdot5$, so $E_i/\lambda' E'$ will usually be very much less than 1. Eqn. (28) may therefore be expressed in the approximate form:

$$T' \simeq 8\cdot85 \times 10^{-12}\frac{\lambda' + \epsilon_2 - 1}{\lambda'\sigma_2}\frac{E_i}{E'} \quad . \quad . \quad . \quad (29)$$

The corresponding approximate expression for the maximum discharge repetition rate, f (discharges/sec), is

$$f \simeq 1\cdot13 \times 10^{11}\frac{\lambda'\sigma_2}{\lambda' + \epsilon_2 - 1}\frac{E'}{E_i} \quad . \quad . \quad . \quad (30)$$

When the cavity is laminar in shape, λ' tends to infinity, and the number of discharges per second is given simply by

$$f \simeq 1\cdot13 \times 10^{11}\sigma_2 E'/E_i \quad . \quad . \quad . \quad . \quad (31)$$

(3.8) Numerical Values

As an example of the application of eqn. (28), the maximum repetition rate of discharges in an oblate spheroidal cavity in polythene will be considered. For polythene, $\epsilon_2 = 2\cdot3$ and, assuming a maximum working temperature of 70° C and a maximum applied stress of 150 kV/cm, the volume conductivity, σ_2, is unlikely to exceed 5×10^{-15} mho/m. Curves showing the relationship between the maximum discharge repetition rate, calculated assuming these values of ϵ_2 and σ_2, and the ratio of applied stress to inception stress, E'/E_i, are given in Fig. 5 for a range of cavity shapes.

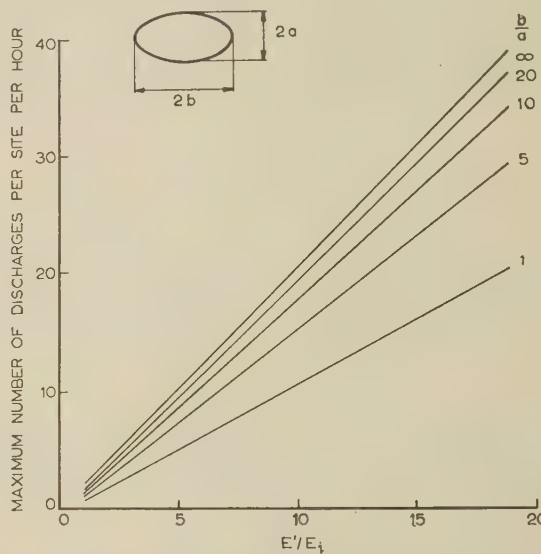


Fig. 5.—Calculated values of maximum repetition rates of discharges in oblate-spheroidal cavities in polythene.
$\epsilon_2 = 2\cdot3$; $\sigma_2 = 5 \times 10^{-15}$ mho/m

Under uniform-field conditions, the breakdown strength of a gas bounded by dielectric surfaces is the same, within close limits, as for breakdown between metal electrodes,[14] and the discharge inception stress, E_i, depends only on the pressure, temperature and composition of the gas in the cavity, and on the maximum depth of the cavity in the field direction. Assuming that the d.c. inception stress of the gas, E_i, is equal to the peak a.c. value given by the results of Hall and Russek,[14] and that the cavity is filled with air at atmospheric pressure and at a maximum temperature of 70° C, then E_i will not be less than 35 kV/cm. In a polythene-insulated power cable, the working stress, E', is unlikely to exceed greatly 150 kV/cm, so that the maximum likely value of E'/E_i is $4\cdot3$. From Fig. 5, it is clear that the number of discharges per discharge site per hour would not exceed 10.

This repetition rate is less, by several orders of magnitude, than would be expected under a.c. conditions, since with a 50 c/s stress only just in excess of the discharge inception value, the rate would be at least two discharges per cycle, or $3\cdot6 \times 10^5$ per hour.

(3.9) Effect of Ripple Voltages

Up to this point it has been assumed that the dielectric is subjected to a pure d.c. stress. Under working conditions, however, the stress in a d.c. power cable is likely to contain an appreciable ripple component, and if this component exceeds the discharge inception value, recurrent discharges will occur in each half-cycle of the ripple waveform, as under a.c. conditions. The level of ripple that can be tolerated must therefore be considered.

The discharge inception stress of a cavity filled with air at atmospheric pressure and at a maximum temperature of 70° C would be not less than 25 kV(r.m.s.)/cm, which corresponds to a minimum stress in the polythene of 11 kV (r.m.s.)/cm, or

0 kV/cm (peak-to-peak). Hence, assuming a d.c. working stress of 150 kV/cm in the polythene, a ripple voltage of at least 20% (peak-to-peak) would be required to cause discharges at the ripple frequency.

Even if the ripple voltage is insufficient to cause discharges at the ripple frequency, it will increase the discharge repetition rate, since a ripple component is added to the d.c. stress in the cavity. For a discharge to occur, the d.c. component has to build up, not to the full inception value, as assumed in Section 3.7, but to the inception stress less the peak value of the ripple stress. The latter is given, for laminar cavities, by $\epsilon_2 E_R$, where E_R is the peak ripple stress in the dielectric. From eqn. (31) it is therefore clear that, with laminar cavities, the effect of ripple is to increase the discharge repetition rate by a factor of $E_i/(E_i - \epsilon_2 E_R)$. If, as is likely, the ripple voltage waveform is non-symmetrical with respect to polarity, E_R must be taken as the peak value of the ripple stress in the dielectric corresponding to voltage half-cycles of the same polarity as the d.c. stress.

(4) EXPERIMENTAL DETERMINATION OF THE DISCHARGE REPETITION RATE

(4.1) Method of Discharge Detection

The usual electrical methods of discharge detection, which depend on detecting the incremental change of voltage across a sample caused by an internal discharge, were found to be unsatisfactory, because of the difficulty of ensuring adequate suppression of electrical interference. An alternative method was therefore developed, which depends on the use of a photomultiplier tube to detect the light emitted by the discharges. This method can be used only to detect discharges in cavities in translucent dielectrics, such as polythene, but has the advantage that the effects of both electrical interference and spurious discharges at electrode edges, etc., can be eliminated.

The samples used in these tests were made of three sheets of polythene, each of 0·005 cm nominal thickness, placed face to face. A cleanly punched circular hole of $\frac{1}{16}$ in diameter at the centre of the middle sheet provided an artificial cylindrical cavity within the sample.

A sectional elevation of the test cell, with a sample in position, is shown in Fig. 6. The upper high-voltage electrode consists of a brass cylinder of $1\frac{1}{4}$ in diameter, with radiused edges, which is embedded in Araldite resin to prevent discharges under the edges. The lower earthed electrode is transparent and is made by vacuum evaporation of a thin layer of lead oxide, followed by a layer of gold, on to the surface of a glass plate.[15] The sample and electrodes are supported on a rubber sheet of $\frac{1}{4}$ in thickness and lightly clamped to the lid of a lightproof box that contains the photomultiplier tube. A hole of $\frac{1}{2}$ in diameter in the rubber sheet and in the lid of the box permits light emitted by discharges in the cavity in the sample to reach the photomultiplier tube, and a mask of opaque paper, punched with a hole of the same diameter as the cavity and aligned with it, ensures that only light produced in the cavity is detected. Discharges in air films trapped between the sample and the electrodes were completely suppressed by placing a drop of silicone fluid between the lower face of the sample and the transparent electrode, and by painting the upper surface of the sample in the region of the cavity with Aquadag.

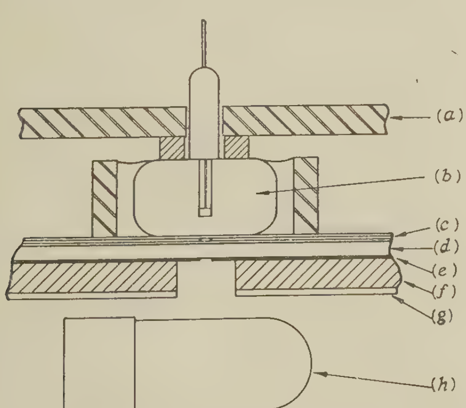


Fig. 6.—Sectional elevation of the test cell for the measurement of discharge repetition rates.

(a) Tufnol clamping strip.
(b) Brass h.v. electrode.
(c) Sample.
(d) Glass plate with transparent electrode on upper surface.
(e) Mask.
(f) Rubber sheet.
(g) Lid of lightproof box.
(h) Photomultiplier tube.

When a discharge occurs in the cavity of a sample, the light emitted causes a voltage pulse from the photomultiplier tube, which is amplified and used to actuate a simple counter circuit. The amplifier gain was adjusted so that the counter just failed to respond to noise pulses. The system was stable and unaffected by mains interference, and in tests of several hours' duration on blank samples, i.e. samples containing no punched cavity, no counts were recorded. The recovery time of the counter was about 0·5 sec. The direct voltage was stabilized to within ±1%, and the 50 c/s ripple component, at 10 kV d.c. output, was 0·3%.

(4.2) Test Procedure

After mounting the sample in the test cell, the testing voltage, V_T, was applied, and the number of discharges recorded by the counter was noted at regular intervals. Each test was continued for about a week.

In the theoretical work it was assumed that the cavity discharged completely and uniformly. However, in cavities of the dimensions used in the tests, it seemed probable that several discharge sites would be established, and in order to permit comparison of the measured repetition rates with the theoretical values it was necessary to determine the number of these sites. This was done with each sample, at the end of the test, as follows. The voltage was raised from zero to the testing value, V_T, in an interval of a few seconds, and the number of discharges caused by this voltage change was counted by observing the amplified signal from the photomultiplier tube on an oscillograph. Since the interval was very short compared with the time required for the steady-state d.c. field to be established, the potential distribution in the sample was similar to that obtained by applying a 50 c/s voltage. Had the cavity discharged completely and uniformly as supposed in the theoretical work, the number of discharges caused by the rapid application of a voltage, V_T, would have been N, the largest integer less than (V_T/V_i), where V_i is the peak 50 c/s discharge inception voltage. In fact, the number of discharges observed, N', was always greater than N, and the number of discharge sites was taken as the integer nearest to (N'/N).

(4.3) Conductivity Measurements

The volume conductivity of the polythene sheet, σ_2, was measured at room temperature and at a number of applied stresses by means of a vibrating-reed electrometer method. In all the tests, the measured conductivity decreased rapidly with time and reached a sensibly steady value after a period of about a day. A typical curve showing the variation with time of the mean conductivities of five samples tested at a stress of 330 kV/cm is given in Fig. 7. The mean steady-state conductivities at stresses of 300 and 600 kV/cm (the stresses used in the discharge-

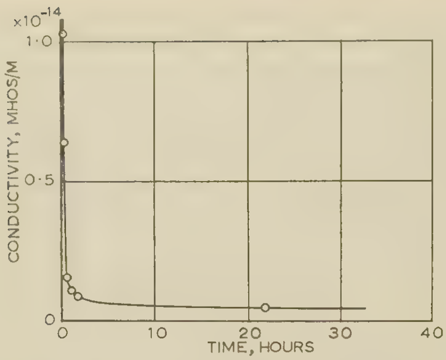


Fig. 7.—Variation of the volume conductivity of polythene with time of application of stress (330 kV/cm).

Each point represents the mean of 5 individual results.

repetition-rate tests) were $3 \cdot 67 \times 10^{-16}$ and $1 \cdot 20 \times 10^{-15}$ mho/m, respectively.

(4.4) Results

Discharge-repetition-rate tests were made on six samples at a stress of 600 kV/cm and on four samples at a stress of 300 kV/cm. With each sample, the discharge repetition rate decreased with time and reached a sensibly steady value after a period ranging from one to three days. A typical curve showing the variation with time of the discharge repetition rate of a sample tested at 300 kV/cm is given in Fig. 8, and the steady-state repetition rates of all the samples are given in Table 1. The diameter of the cavities (0·159 cm) was large compared with their depth (0·005 cm), and so the cavity shape was approximately laminar. The theoretical maximum discharge repetition rates given in Table 1 were therefore calculated from eqn. (31) using the measured values of conductivity given in Section 4.3, and assuming a value for E_i, the discharge inception stress of the air in the cavity, of 136 kV/cm.[14]

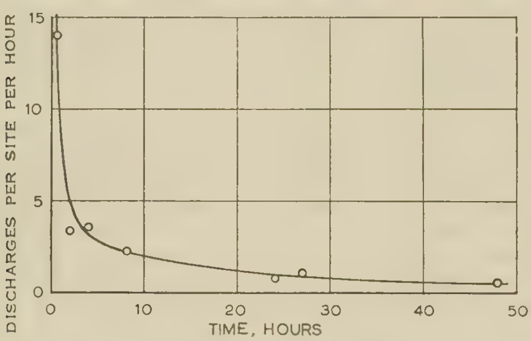


Fig. 8.—Variation of the repetition rate of discharges in a cavity in polythene with time of application of stress (300 kV/cm).

Table 1

Comparison of the Measured Steady-State Discharge Repetition Rates of Polythene Samples with the Calculated Maximum Values

Sample No.	Stress in dielectric	Estimated number of discharge sites	Steady-state discharge repetition rate		Calculated maximum repetition rate
			per cavity	per site	
	kV/cm		Discharges/hour		Discharges/site/hour
1	600	6	5·0	0·8	2·18
2		5	9·0	1·8	
3		4	2·5	0·6	
4		2	1·3	0·6	
5		3	2·0	0·7	
6		3	1·4	0·5	
7	300	3	0·3	0·10	0·34
8		3	0·3	0·10	
9		2	0·3	0·15	
10		4	0·2	0·05	

The theoretical maximum repetition rates are calculated assuming that the cavity discharges through a single channel, and that the residual voltage across the cavity after completion of a discharge is negligible. The latter assumption is probably justified,[16] but the measurements described in Section 4.2 showed that, with all the samples, the cavity contained several discharge sites. The number of discharges per hour per cavity could therefore exceed the calculated maximum rate, as was the case with Samples 1, 2 and 3. However, if the cavity discharges through N channels instead of a single channel, the number of discharges per hour per cavity, at a first approximation, would be expected to be increased by a factor of N, and the theoretical rate given by eqn. (31) may then be regarded as the maximum possible repetition rate per discharge site. The measured repetition rates per discharge site are all less than the theoretical maximum values, and though this is not claimed to provide complete verification of the theoretical work, it is clear that the experimental and theoretical results are at least consistent.

(5) ACCELERATED LIFE TESTS

In order to estimate the effect of a cavity on the d.c. life of a dielectric, it is necessary to know not only the discharge repetition rate but also the number of discharges in the cavity which the dielectric can withstand. If the number of discharges required to bring about failure is not greatly influenced by the frequency of the discharges, the latter information can be obtained in a relatively short time by means of 50 c/s life tests. Care is needed in the design of these tests, since the discharge characteristics can change with time, and, in particular, changes in the conductivity of the cavity walls, caused by the discharges themselves, can result in the discharges becoming temporarily extinguished or intermittent.[17] However, discharges in laminar cavities adjacent to a conductor (such as could be produced in a cable by failure of the adhesion between the dielectric and the conductor) are unlikely to become extinguished in this way, and it is therefore preferable to carry out accelerated tests with specimens containing cavities of this kind. The damaging effect of a discharge probably depends on the stress in the dielectric on which it impinges, as well as on the discharge magnitude, so that, in the accelerated tests, the samples are subjected to an a.c. stress of peak value equal to the intended d.c. working value.

Tests with 50 c/s voltages on specimens of Alkathene grade 0·3 with 0·1% Nonox W.S.P. antioxidant, containing cavities of $\frac{1}{8}$ in diameter and of up to 0·1 mm in depth, which were adjacent to a copper conductor, have shown that at a temperature of 65° C a life of at least 600 hours is obtainable with peak stresses in the polythene in excess of 150 kV/cm. The stresses were in the region of two to three times the discharge inception stress of the specimens, so that under the stated conditions of test this particular polythene compound is capable of withstanding at least 2×10^8 discharges without failure. It was shown in Section 3.8 that, under likely working conditions in a polythene-insulated h.v. d.c. cable, the steady-state number of discharges per discharge site per hour would not exceed 10, so that the predicted time to failure caused by discharges is of the order of 10^7 hours, or very much greater than the normal cable working life of about 40 years.

(6) DISCUSSION

An expression has been derived for the steady-state repetition rate of discharges in a gas-filled cavity in a dielectric subjected to a d.c. stress. The actual repetition rate depends on the surface conductivity of the walls of the cavity, which will in general be unknown, but it has been shown that the repetition rate is a maximum when the surface conductivity is zero, and that this maximum rate depends only on the permittivity and volume conductivity of the dielectric, the ratio of the stress in the dielectric to the breakdown stress of the gas and the shape of the cavity, all of which are known, or measurable, quantities. In this work a number of simplifying assumptions were made, the consequences of which will now be considered.

The volume conductivity of the dielectric was assumed to be ohmic, whereas the conductivity of most dielectrics decreases with time of application of stress and tends to a steady-state value which increases with increasing stress. The time dependence causes the discharge repetition rate to exceed the estimated steady-state value for the first few hours after application of stress. When, however, the conductivity in the bulk of the dielectric has attained the steady-state value, the time dependence is of consequence only in the region adjacent to the cavity, where the conductivity is prevented from attaining the steady-state value by the recurrent changes of stress caused by the discharges. After completion of a discharge the cavity is presumably surrounded by a region of increased conductivity, which tends partially to short-circuit the cavity, and so results in the discharge repetition rate being lower than if the conductivity were ohmic. The effect of the stress dependence of the steady-state conductivity would be to reduce the non-uniformity of the stress distribution in the dielectric, but in the region adjacent to the cavity, where the stress is non-uniform, the conductivity does not attain its steady-state value, so that the effect is probably of less importance than the time dependence. The effect of a non-ohmic conductivity is therefore likely to be to decrease the discharge repetition rate, and so the expressions for the maximum rate, given by eqns. (28)–(31), do not need correction.

The surface conductivity was also assumed to be ohmic and to have a non-uniform distribution. However, any distribution of surface conductivity partially short-circuits the cavity and so reduces the discharge repetition rate. The result that the repetition rate is a maximum when the surface conductivity is zero is therefore likely to be generally valid.

The cavity was assumed to discharge uniformly, although in a large cavity several discharge sites would probably be established. If all the sites discharged simultaneously, the calculated maximum rate would apply to each site. The interval between discharges is controlled by the rate of arrival of charge at the cavity surface, which is a maximum if the cavity discharges completely, since this produces the maximum convergence of the lines of force. If the sites do not discharge simultaneously, therefore, the repetition rate at each site will still not exceed the calculated maximum value.

Finally, it was assumed that the residual voltage across the cavity after completion of a discharge is negligibly small compared with the breakdown voltage. The effect of a non-negligible residual voltage would be to increase the discharge repetition rate above the calculated maximum value, though the energy dissipated in each individual discharge would be correspondingly less. However, Mason's results[16] indicate that the residual voltage is small, and the consistency of the measured and calculated values of the repetition rate of discharges in polythene samples (see Section 4.4) provides further evidence that the assumption is justified. It is concluded that the expression given by eqns. (28)–(31) may be used with reasonable confidence to estimate the maximum steady-state repetition rates of discharges in cavities in dielectrics subjected to d.c. stresses.

Direct measurements of the repetition rate of discharges in cavities in polythene, described in Section 4, have confirmed that for several hours after the application of stress the rate may considerably exceed the steady-state value, and comparison of Figs. 7 and 8 suggests that this time dependence is at least qualitatively explicable in terms of the similar variation of the volume conductivity, which is the result of dielectric absorption. Most low-loss dielectrics show absorption to varying degrees,[18] and so this time dependence of the discharge repetition rate is likely to be a general effect. Since the direction of power flow through a d.c. link between two a.c. power systems may be conveniently controlled by reversing the polarity of the voltage on the cables, this effect could be of practical importance. If, as may be the case with the proposed cross-Channel cable, the polarity is reversed several times a day, the average discharge repetition rate could considerably exceed the steady-state value, with a possible deleterious effect on the dielectric life.

The discharge repetition rate could be increased by ripple voltages, and with polythene dielectric, for example, subjected to a d.c. stress of 150 kV/cm and containing laminar cavities with a discharge inception stress, E_i, of 35 kV/cm, the steady-state repetition rate would be doubled by a ripple stress in the dielectric of 7·6 kV/cm. If the ripple waveform is symmetrical with respect to polarity, this corresponds to a 10% peak-to-peak voltage ripple. With the same assumptions, a 20% peak-to-peak voltage ripple would be required to cause discharges at the ripple frequency.

Under d.c. conditions, there is no sudden onset of discharges at a critical stress, as at 50 c/s, and although the d.c. discharge-inception stress could be defined as the stress at which the discharge repetition rate tends to zero, it would clearly not be directly measurable. It is therefore preferable to detect cavities in insulation intended for d.c. use by means of an a.c. test. With dielectrics such as polythene, methods for the continuous measurement of the 50 c/s discharge-inception voltage on long lengths of cable core are readily available,[19] and if the cavities are supposed to be of laminar shape and adjacent to the conductor, the maximum steady-state discharge repetition rate under d.c. working conditions may be calculated from the volume conductivity of the dielectric and the inception stress of the gas in the cavities, which may in turn be derived from the 50 c/s discharge-inception voltage. This has been done in Table 2 for a 100 kV d.c. cable designed to operate at a maximum conductor temperature of 65° C and a conductor stress of 150 kV/cm (computed on the assumption of a hyperbolic distribution). The radial depth of the cavities, assuming them to be adjacent to the conductor, may also be determined from the discharge-inception voltage, using published data,[14] and estimated values are given in the Table.

Table 2

Discharge Repetition Rates in Polythene Cables

50 c/s discharge inception voltage	Maximum repetition rate	Radial depth of conductor adjacent cavity
kV(r.m.s.)	Discharges/site/hour	mm
10	6·2	0·70
15	4·1	0·23
20	3·1	0·10
25	2·5	0·07

Volume conductivity, 5×10^{-15} mho/m.

It will be seen that decreasing values of the 50 c/s inception voltage are accompanied by a comparatively slow increase in the maximum discharge repetition rate, but by a rapid increase in the cavity size and hence in the energy dissipated in individual discharges. Hence, as the discharge-inception level falls, the dielectric life is likely to decrease very much more rapidly than might be supposed from the increase in the discharge repetition rate. However, in the example chosen, it would seem sufficient to specify a minimum discharge-inception voltage of 20 kV to ensure adequate life under the d.c. operating conditions.

From considerations of electrical properties, and of the dielectrics discussed, impregnated paper and polythene are the most suitable for use in h.v. d.c. cables. Most of the operating experience to date has been obtained with impregnated paper, and hence, at the present time, this dielectric is perhaps the most favoured. For submarine applications, polythene might be advantageous, since it could probably be used without a watertight sheath, and also has an appreciably lower thermal resistivity. The d.c. strength is adequate to permit operation at mean stresses in the region of 150 kV/cm, and estimates of the discharge repetition rate, together with the results of accelerated life tests, have shown that at these stresses, discharges in cavities are unlikely to be a serious problem. Frequent reversals of polarity, or unusually high levels of ripple voltage, could increase the discharge repetition rate and so shorten the dielectric life, although under likely working conditions the factor of safety should be adequate, provided that the arduous handling conditions to which submarine power cables are inevitably subjected do not cause excessive damage to the dielectric. This aspect could be resolved only by practical trials.

(7) ACKNOWLEDGMENTS

Acknowledgment is due to Dr. L. G. Brazier, Director of Research and Education, British Insulated Callender's Cables Ltd., for permission to publish the paper. The authors also wish to thank their colleagues in the Physics Department who assisted with the experimental work.

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(9) APPENDIX

The Maximum Discharge Repetition Rate

The interval between successive discharges, T, is given by

$$T = -\tau \log_\varepsilon (1 - E_i/E'\lambda) \quad . \quad . \quad . \quad (32)$$

where

$$1/\lambda = 1 - (1/\sigma_2)(\alpha \operatorname{arc\,cot} \alpha - 1) \times [(\sigma_{s0}\alpha/c) + (\sigma_1 - \sigma_2)(1 + \alpha^2)] \quad (33)$$

and

$$\tau = \frac{\epsilon_0(\epsilon_1 - \epsilon_2)(1 + \alpha^2)(\alpha \operatorname{arc\,cot} \alpha - 1) - \epsilon_0\epsilon_2}{[(\sigma_{s0}\alpha/c) + (\sigma_1 - \sigma_2)(1 + \alpha^2)](\alpha \operatorname{arc\,cot} \alpha - 1) - \sigma_2} \quad (34)$$

The least value of T is to be determined, when σ_{s0} is allowed to take any value greater than zero. Differentiating T with respect to σ_{s0} gives

$$\frac{\partial T}{\partial \sigma_{s0}} = -\frac{\partial \lambda}{\partial \sigma_{s0}} \frac{\tau}{\lambda} \frac{E_i}{\lambda E' - E_i} - \frac{\partial \tau}{\partial \sigma_{s0}} \log_\varepsilon (1 - E_i/E'\lambda) \quad (35)$$

But, from eqns. (30) and (31), $\tau = u\lambda$, where u is independent of σ_{s0}, so that

$$\frac{\partial \tau}{\partial \sigma_{s0}} = \frac{\tau}{\lambda} \frac{\partial \lambda}{\partial \sigma_{s0}} \quad . \quad . \quad . \quad . \quad . \quad (36)$$

and

$$\frac{\partial T}{\partial \sigma_{s0}} = -\frac{\partial \tau}{\partial \sigma_{s0}} \left[\frac{E_i}{\lambda E'} \left(1 - \frac{E_i}{\lambda E'}\right)^{-1} + \log_\varepsilon \left(1 - \frac{E_i}{\lambda E'}\right) \right] \quad (37)$$

For discharges to occur, the steady-state field in the cavity, $\lambda E'$, must exceed E_i. Put $E_i/\lambda E' = \psi$: then $0 < \psi < 1$ and

$$\begin{aligned} \frac{\partial T}{\partial \sigma_{s0}} &= -\frac{\partial \tau}{\partial \sigma_{s0}} [\psi(1 - \psi)^{-1} + \log_\varepsilon (1 - \psi)] \\ &= -\frac{\partial \tau}{\partial \sigma_{s0}} \left[(\psi + \psi^2 + \psi^3 \ldots) - \left(\psi + \frac{\psi^2}{2} + \frac{\psi^3}{3} \ldots\right) \right] \\ &= -\frac{\partial \tau}{\partial \sigma_{s0}} \left(\frac{\psi^2}{2} + \frac{2\psi^3}{3} + \frac{3\psi^4}{4} \ldots \right) \end{aligned}$$

The expression in brackets is greater than zero for all σ_{s0}. The sign of $\partial \tau / \partial \sigma_{s0}$ must now be considered:

$$\frac{\partial\tau}{\partial\sigma_{s0}} = \frac{\alpha(\alpha \operatorname{arc\,cot} \alpha - 1)}{4\pi c} \times \frac{\epsilon_0(\epsilon_2 - \epsilon_1)(1 + \alpha^2)(\alpha \operatorname{arc\,cot} \alpha - 1) + \epsilon_0\epsilon_2}{\{[(\sigma_{s0}\alpha/c) + (\sigma_1 - \sigma_2)(1 + \alpha^2)](\alpha \operatorname{arc\,cot} \alpha - 1) - \sigma_2\}^2} \quad (38)$$

$(\alpha \operatorname{arc\,cot} \alpha - 1)$ is less than zero for all α, and $(1 + \alpha^2)(\alpha \operatorname{arc\,cot} \alpha - 1)$ has a minimum value of -1. Hence $\partial\tau/\partial\sigma_{s0}$ is less than zero for all σ_{s0}, and from eqn. (35) it is clear that $\partial T/\partial\sigma_{s0}$ is greater than zero for all σ_{s0}, so that T has its least value when $\sigma_{s0} = 0$.

DISCUSSION BEFORE THE SUPPLY SECTION, 13TH JANUARY, 1960

Mr. R. Davis: The scope of the paper is wider than its title would suggest, covering as it does an examination of the desirable properties of a cable for direct voltages, and narrower in that the main experimental study is restricted to one cable dielectric. Nevertheless, the authors treat their main topic both theoretically and experimentally in a skilful and elegant way and confirm that internal discharges are not a serious obstacle to the operation of cables under direct stresses.

The theoretical treatment of the problem leads to a very simple formulae for the discharge repetition frequency [eqn. (31)]; Fig. 5 shows that the more elaborate formulae taking cavity shapes into account do not change the result by more than a factor of 2, and the experimental confirmation of the theory can be regarded as satisfactory. In Section 9 the authors show that the discharge rate is a maximum when the surface conductivity of the cavity is zero; this can be shown by the extension of simple ideas. If the conductivity is infinite, no charge can build up on the surface and no field can exist in the cavity; if the conductivity is finite, charge formed on the surface can leak away at a rate which decreases with decreasing conductivity, the rate being a minimum when the conductivity is zero.

By observing the light associated with a discharge in a cavity, the authors have developed a useful method for determining the discharge frequency. While the method is applicable only to translucent dielectrics it has the advantage that the observations can be restricted to a small region, so that the need for restricting corona discharge in other parts of the circuit ceases to be of paramount importance. By the use of photomultipliers and amplifiers the method can presumably be made extremely sensitive, and it would be of interest to know whether the technique permits discrimination between discharges of different severities.

Mr. C. G. Garton: The paper makes a useful contribution to the theory of discharges. The mathematical solution has long been wanted by those working in this field.

One point in the solution requires further consideration. The surface conductivity is defined in terms of a constant value at the pole of the cavity, and decreases toward the equator, where it is smaller in the ratio of the minor to the major axis. This implies that, when the ratio goes to zero to represent a laminar cavity, the surface conduction path is open-circuited at the periphery of the cavity. This is the position where, intuitively, one expects the conductivity to have most effect. The authors might compare their present result with one obtained by assigning a given conductivity at the equator, and letting it go to infinity at the poles (assuming that the equations still lend themselves to solution under these conditions).

More information might be given on the sensitivity of the detector. The authors believe that they were not recording spurious pulses, but they leave open the question whether they were recording all the real ones. Their adjustment of the counter so that it 'just failed to respond to noise pulses' leaves open the question of its sensitivity, unless one knows the level of random noise for their experimental conditions. In general, it is difficult to keep the noise level below that of the smallest discharges.

The optical method of detection is elegant, but, of course, fails except for the few translucent dielectrics. It is, however, possible with a doubly screened room and adequate filtering to reduce the noise level in a normal type of discharge detector to the point where occasional d.c. discharges can be recorded over periods of hours without spurious counts. My colleague, Mr. Krasucki, has such a system in operation, recording finally on a Dekatron counter.

The authors might have emphasized more strongly the importance of large voids. In cables, unless they are monitored continuously during manufacture, the discharge of an occasional large void cannot always be distinguished from those of numerous small ones. Yet where a large enough void is present, the safe working stress is likely to be decreased from the value of 150 kV/cm mentioned by the authors to something less than 50 kV/cm.

Mr. C. C. Barnes: Section 2.2.3 refers very briefly to Terylene and polystyrene, but dismisses them because of high cost. Costs are always changing, and technical details on these materials would be a useful addition to the paper.

The relatively low impulse breakdown strength of polythene, given as 700–1 000 kV/cm on thick-walled cables at 20° C, compares favourably with some tests on paper cables. The range is wide, but I assume this is because results have been quoted from tests made under differing conditions in various countries. For the cross-Channel cable scheme now under construction between the British and French power systems, solid-type paper-insulated cables will be used. Consideration was given to the possibility of using polythene-insulated cables, but problems associated with the manufacturing technique and the possibility of frequent polarity reversals introduced hazards which could not be accepted, bearing in mind the high costs associated with any repairs and the long outage which can result if service failures occur. The authors have been very cautious in their observations regarding the use of polythene cables for submarine projects. They state that under likely working conditions the safety factor would be adequate. What exactly do they mean when referring to a safety factor applied to submarine cables and what value would they regard as satisfactory for such projects?

Rather than starting with established materials, such as impregnated paper, considering standard forms of cable construction and then proceeding to explain why these materials have limitations, would not a more realistic approach be for chemists and physicists to work closely together to produce a finished product which would provide a predetermined range of characteristics?

Mr. E. L. Davey: I would immediately increase the figures given in Section 1 for the permissible length of a cable for a.c. transmission by 25% for existing lower-permittivity dielectrics and would then double the resultant figures by the use of reactive compensation apparatus at one end of the feeder to reduce the charging-current effects on rating by the maximum amount of 50%. Generally speaking, the limit to the length of cable with a.c. transmission is set by economics, and provided that the power is sufficient this is of the order of 25 miles of 132 kV cable. The power must be sufficient to make d.c. cable a reasonable proposition—something of the order of 400 MW—since the cost of d.c. equipment varies roughly as the square root of the power.

It is stated that pressurization is not so advantageous under d.c.

conditions. The direct ionization inception voltage increases and the discharge rate decreases with pressure, but an increase of stress rating would mean that the region of intrinsic material breakdown would be entered. However, it is important not to allow vacuous conditions to exist in the gas spaces since, as recorded in Section 2.2.4, lower breakdown values are obtained in stability tests.

On the matter of stress inversion the importance of limiting the stress to a maximum at all points in the cable dielectric lies in the risk of discharge in voids. An important deduction can be made from the curves in Fig. 1, namely that there are two limitations to the temperature—the drop between conductor and sheath and the maximum temperature to which the dielectric can be subjected.

Mr. R. C. Mildner: The authors imply that the radial stress distribution can be calculated by the classical theory. Gorodetzky,* however, made direct measurements of the radial potential distribution. He found quite remarkable discrepancies in the calculated values amounting in some cases to more than 300% at the boundaries. These he attributed to space charges.

Humphries and I (Reference 10) deduced the discharge repetition period by a comparison of the life of polythene cables under d.c. stress with that under comparable a.c. conditions. We found that the period was of the order of minutes or hours, as compared with the periods of weeks and months that had been previously suggested. The more refined theory of the authors provides a remarkable confirmation of these rates, especially so since we were concerned with full-sized cables operating at stresses of 300 kV/cm and more, and working under load cycles of up to 70° C. Mason,† in his work on a.c. discharges in cavities, has pointed out that, initially in a cavity whose diameter is larger than the discharge site, one observes a series of discharges in which the sites are located apparently at random over the surface of the cavity; and that after a period the discharges are confined to a few cavities, then ultimately to one cavity, which may eventually develop into a breakdown. Did the authors re-check the number of discharge sites and the discharge initiation voltage at the end of their d.c. run?

In the discussion on the increase in the rate of discharge repetition which occurs when direct voltage is first applied, it is interesting that, with polythene, as 70° C is approached the dielectric absorption has almost completely disappeared, so that this increase in the rate will be less important at higher temperatures where the discharge resistance tends to be somewhat lower.

In Section 6 the authors suggest that the energy in the discharge increases with the size of the cavity. I disagree. It is very rarely true in the case of high-quality polythene-insulated cable, i.e. cables in which the voids will not be greater than 2 mils in radial depth. We obtained experimental confirmation of this by taking a length of cable and measuring the variation of discharge initiation voltage along the length by means of a travelling electrode. The core was then put in water with the ends above the surface, and an over-voltage two or three times greater than the discharge inception voltage was applied. We invariably found that the breakdown occurred not at the lowest point of discharge initiation voltage, but somewhere else. The significance of this is that, if the cable is operated in the neighbourhood of the discharge inception voltage, this voltage provides a good criterion for the subsequent cable performance to the extent that it is stable with time, but if tested at an over-voltage two or three times greater, even under direct voltages, it will no longer be a good criterion. Eventually we shall find some way of circumventing this difficulty, but at the moment it prevents the discharge measurements from becoming the perfect non-destructive test.

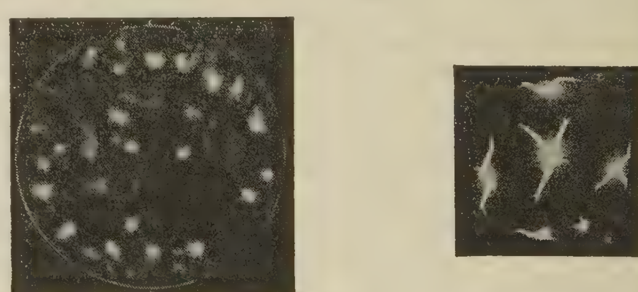
* Gorodetzky, S. S.: '220–400 kV Direct-Current Cables', C.I.G.R.É., Paris, 1958, Paper No. 206.
† Mason, J. H.: 'The Deterioration and Breakdown of Dielectrics resulting from Internal Discharges', *Proceedings I.E.E.*, Paper No. 1053, January, 1951 (98, Part I, p. 44).

Dr. B. Salvage: A greater emphasis on the experimental approach to the problem to balance the valuable theoretical work that has been done would have been helpful. In particular, further information on the effect of the electric stress and data on the effect of the temperature on the discharge repetition rate would be interesting.

The authors have found that the discharge repetition rate decreased very considerably at the start of a discharge test and they have attributed this to the decrease in the volume conductivity of the dielectric in the region of the cavity during the initial part of the test. There is now considerable evidence that the self-extinction effect of discharges in cavities (see Reference 17) is caused by an increase in the conductivity of the surfaces of the cavities, owing to the formation of a semiconducting deposit on the surfaces. Could not a similar effect have occurred in the experiments which the authors have described in the present paper, resulting in a reduction in the discharge repetition rate?

Have the authors carried out any experiments to follow the degradation processes in the region of the dielectric surrounding a discharging cavity? They have assessed the probable life of dielectrics subjected to high direct electric stresses by reference to tests with alternating stresses and it is important to know whether the degradation processes are similar under the two conditions.

Dr. J. H. Mason: In Section 5 the authors suggest that discharges in cavities adjacent to an electrode are unlikely to be temporarily extinguished by the formation of semiconducting films. E.R.A. research shows, however, that discharges propagating from such channels are sometimes short-circuited by carbonization, whereas discharges in cavities entirely enclosed in polythene cause slower penetration, but continue until failure occurs.* In the same paper, it was shown that discharges in small cavities in polythene occur at numerous sites, each less than 0·5 mm diameter, as in Fig. A(i). Subsequent work



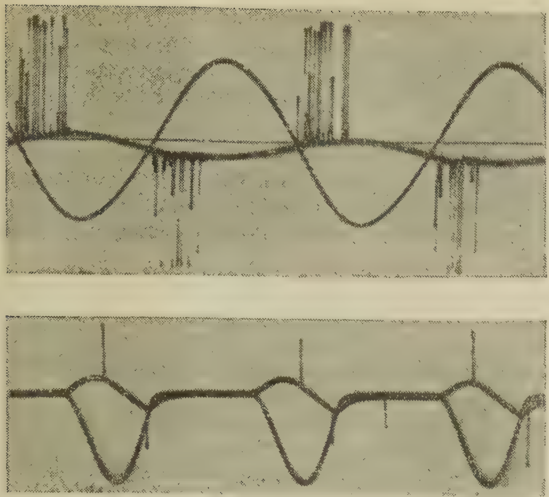
(i) (ii)

Fig. A.—Electrophotographs showing discharge sites.

(i) In cavity 5 mm diameter and 0·3 mm depth between polythene and a photosensitive surface. 5 cycles at 3 kV (r.m.s.) applied.
(ii) In cavity 4 mm diameter between two photographic plates, spaced 0·8 mm apart. 7 kV direct voltage applied for 0·04 sec.

showed† that each discharge site was of irregular shape, as in Fig. A(ii). Evidently the factors governing the rate of charge leakage from any discharge site must be very complex, and I wonder whether the authors' elaborate mathematical analysis was justified. If the volume resistivity of polythene is 10^{17} ohm-cm, the discharge time-constant would be some 23 000 sec, i.e. less than 0·05 discharges per hour would occur

* Mason, J. H.: 'The Deterioration and Breakdown of Dielectrics resulting from Internal Discharges', *Proceedings I.E.E.*, Paper No. 1053, January, 1951 (98, Part I, p. 44).
† Mason, J. H.: 'Breakdown of Insulation by Discharges', *Proceedings I.E.E.*, Paper No. 1471 M, 1953 (100, Part IIA, p. 149).



(i)

(ii)

Fig. B.—Discharges in a cavity 2·5 mm diameter and 0·2 mm depth in polythene.
(i) A.C. 8 kV (peak) ($2V_i$).
(ii) Pulses, 8 kV (peak).

at each site. This value is smaller than observed (see Table 1) but would not conflict with the conclusion that discharges should cause negligible damage when polythene is subjected to constant d.c. stress.

The danger of polarity reversals should however be given greater emphasis. Comparison of Figs. B(i) and (ii) shows that many more discharges occur with alternating voltage than with uni-directional pulses. With a working voltage of $5V_i$, polarity reversal four times a day, together with the effect of dielectric absorption, might raise the number of discharges fifty-fold compared with constant direct current. Thus, for operation with polarity reversals it would seem advisable to use discharge-free cable. Is it possible to achieve this by screening the conductors, or will it be necessary and economic to apply high gas pressure in the inner and outer conductors?

Mr. J. E. L. Robinson: Has any attempt been made to check by other means the deduced number of sites per void? Does a single discharge cause discernible marking which can be examined optically or otherwise? Comparison of such a record with the findings of the paper could provide valuable corroboration of the theory.

Is it true that, although in principle the repetition frequency depends on the size of the void, the degree of dependence is slight and justifies the size parameters disappearing in the simplified equations?

Again, the most important practical case referred to in the paper is that in which one side of the void is the conductor. Have the authors attempted correlation of measurements with the theory in this case?

Mr. G. S. Buckingham: The predicted time of failure of the cable under normal working stress—of the order of 10^7 hours—represents a substantial safety factor. Is it not a little unnecessary to expect a ratio of expectation of life of more than 25 : 1?

In a previous paper* it has been suggested that we might be able to look forward to the time when impregnated paper and polystyrene tapes were interleaved in a lead-covered cable. Such a design would have as its purpose the elimination of gas occlusions, thus increasing the life of the cable. Can the authors give any up-to-date information on the possibility of mixing paper and polystyrene in one cable?

THE AUTHORS' REPLY TO THE ABOVE DISCUSSION

Messrs. E. C. Rogers and **D. J. Skipper** (*in reply*):

To Mr. Davis.—The light emitted by a discharge is presumably closely related to the energy dissipated, and so no difficulty should be experienced in using the optical technique to discriminate between discharges of different severities.

To Mr. Garton.—If, in Section 3.5.2, σ_s is defined in terms of a constant value, σ'_{s0}, at the equator, instead of σ_{s0} at the poles, the interval between discharges, given by eqn. (27), is unchanged except that in the expressions for τ and $1/\lambda$ [eqns. (21) and (26)] the term $\sigma_{s0}\alpha/c$ must be replaced by $\sigma'_{s0}(1+\alpha^2)^{1/2}/c$. The interval is a minimum when $\sigma'_{s0}=0$, and so the expressions for the maximum repetition rate, given by eqns. (28)–(31), are unaltered.

Observation of the photomultiplier output with a cathode-ray oscillograph showed the discharge pulses to be of considerably greater amplitude than the noise, and there were no pulses of intermediate amplitude between these two levels. We are therefore confident that, with the counter set so as just not to respond to noise, all discharges were detected.

It is normal practice to monitor polythene cable continuously during manufacture, so that occasional large voids should be detected. Production experience shows that, with a cable design similar to that described in Section 6, the requirement for a minimum discharge-inception voltage of 20 kV can be met and the suggested working stress of 150 kV/cm should therefore be attainable.

To Mr. Barnes.—Economic considerations apart, polystyrene and Terylene would tend to be rejected on mechanical grounds. Also, the relatively high thermal resistivity of polystyrene would be detrimental to both stress distribution and power rating.

It is difficult to ascribe a numerical value to safety factor, but the margin of safety allowed was judged adequate from a knowledge of the a.c. life characteristics of polythene cables. The suggested substitution of molecular engineering for cable engineering is felt to be rather more idealistic than realistic at the present time.

To Mr. Davey.—We consider that the 25% increase in maximum permissible length suggested for pre-impregnated cables is not justified solely by permittivity considerations, but must depend to a large extent on the lower working stress of this type of dielectric. The statement that the power must be of the order of 400 MW to make d.c. cable a reasonable proposition conflicts with the conclusions of other published work.†

To Mr. Mildner.—The measurements of Gorodetzky were not confirmed by Oudin and Bele,[9] who claim to have attained close agreement between measured and calculated stress distributions.

The estimated numbers of discharge sites quoted in Table 1 are based on measurements made at the end of the tests. In some instances the initial values were slightly higher. We consider the contention that discharge energy is unrelated to cavity depth to be unproven, since with over-voltage tests of the order mentioned, breakdown mechanisms other than discharge erosion might be operative. Thus channels might be propagated from inclusions or other points of enhanced stress, so that failure would not necessarily occur at the point of lowest discharge-inception voltage.

To Dr. Salvage.—The discharge repetition rate is directly proportional to the volume conductivity of the dielectric, and so it

* THORNTON, E. P. G., and BOOTH, D. H.: 'The Design and Performance of the Gas-Filled Cable System', *Proceedings I.E.E.*, Paper No. 2754 S, October, 1958 (106 A, p. 207).

† LANE, F. J., RATHSMAN, G., LAMM, U., and SMEDSELT, K. S.: 'Comparison of Transmission Costs for High-Voltage A.C. and D.C. Systems', C.I.G.R.É., Paris, 1956, Paper No. 417.

seems reasonable to ascribe the decrease of repetition rate with time of application stress to the similar variation of the conductivity. It is possible, however, that an increase in the surface conductivity of the cavity walls also contributes to the effect.

We agree that, ideally, it is desirable to confirm that the dielectric degradation processes are the same with direct as with alternating current. There is, however, the practical difficulty that life tests on dielectrics subjected to unidirectional discharges would need to be of very long duration, since there would seem to be no means of accelerating the rate of leakage of charge from the surface without interfering with the degradation processes.

To Dr. Mason.—In Section 5, we state that discharges in laminar cavities (i.e. cavities of large diameter/depth ratio) adjacent to an electrode are unlikely to be extinguished by semiconducting surface films. Tests at 50 c/s of several days' duration have shown that with such cavities the discharge-extinction voltage remains sensibly constant, whereas with cavities of small diameter/depth ratio discharges may be extinguished.[17] Under possible working conditions in a polythene-insulated d.c. cable, and with the volume conductivity quoted, the estimated repetition rate of 0·05 discharge per site per hour could be in error by a factor of at least 100. We therefore consider the more refined analysis given in the paper to be amply justified.

Both conductor and dielectric screening would normally be employed, but this would not necessarily guarantee discharge-free operation at 150 kV/cm. However, the suggested effect of polarity reversal is felt to be exaggerated, since as Mr. Mildner rightly points out, dielectric absorption effects in polythene are much reduced at maximum cable-operating temperatures. Thus, even with fairly frequent reversals of polarity, it is not considered necessary to specify discharge-free cable, nor to employ pressure-assisted designs.

To Mr. Robinson.—The deduced number of discharge sites has not been checked by other means, but the technique of electro-photography described by Dr. Mason provides a means of doing this and of recording the occurrence of a single discharge.

The void size (i.e. depth in the field direction) influences the repetition rate only by determining E_i.[14] The dependence on shape, i.e. b/a, shown in Fig. 5 is not great, and for practical purposes the expression for a laminar void [eqn. (31)] may be used, for which shape the rate is a maximum.

No repetition-rate measurements were made with electrode adjacent voids.

To Mr. Buckingham.—We do not suggest that a predicted time to failure of 10^7 hours is a necessary requirement; the point we wish to make is that under the working conditions considered a time of this order is likely to be required for discharges to cause failure. Discharge damage is therefore unlikely to be a serious hazard.

The use of polystyrene tapes interleaved with pre-impregnated paper would result in a very uneven stress distribution with direct voltages, because of the widely differing resistivities of the two materials. The advisability of using such a construction with d.c. cables is therefore open to doubt.

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THE CHARACTERISTICS AND PROTECTION OF SEMICONDUCTOR RECTIFIERS

By D. B. CORBYN, B.Sc.(Eng.), Member, and N. L. POTTER, B.Sc.(Eng.), Graduate.

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SUMMARY

The paper first describes the probable sphere of application of monocrystalline semiconductor rectifiers and states the problems which must be solved to ensure reliable operation under both normal and abnormal conditions. The basic characteristics of these rectifiers are then discussed. A brief description is given of the special test methods required to establish the characteristics, and it is shown that important properties can be entirely overlooked if test methods are inadequate.

Operating conditions are examined under both normal conditions and during faults and overloads, and the protection requirements are deduced.

The properties of h.r.c. fuses are briefly discussed, and by consideration of the rectifier properties and operating conditions the requirements are stated for the special fuses necessary. It is shown that these special fuses provide a basis for a protective system which enables full use to be made of the new semiconductor rectifiers.

The paper concludes with a description of the practical application of these special fuses, and shows how different forms of protection can be co-ordinated to ensure satisfactory operation of semiconductor rectifiers under different conditions for a wide variety of duties.

(1) INTRODUCTION

Technical changes in the design of high-power rectifiers have made a re-examination of protective measures necessary. Until recently, most high-power direct current was supplied by mercury-arc, mechanical or selenium rectifiers, and special protection schemes were evolved for each type of equipment. The mercury-arc rectifier experiences occasional short-circuits (backfires) between anode and cathode, but suffers no permanent harm so long as it can be switched off within a few cycles. The mechanical rectifier, mainly applied for very heavy direct currents in the region of 100–600 volts, is liable to backfire because of sudden changes of supply voltage or faults in the contact mechanism. The standard protection is to short-circuit the rectifier input terminals in less than 2 millisec, before the next contact of the rectifier can open. This prevents more than one contact being involved. Burnt contacts are expendable, and can be replaced in a few minutes. The selenium rectifier is mainly used for small powers and in control circuits. It has rather a low efficiency, but a great ability to withstand momentary over-voltages and over-currents, so that protection is extremely simple, usually involving only standard fusegear and contactors.

The advent of germanium and silicon semiconductor rectifiers has completely altered rectifier protection requirements. These rectifiers are simple and small, and give a uniformly high efficiency over a wide range. However, they are susceptible to damage by over-currents, owing to their small thermal mass, and to damage by over-voltages, owing to breakdown phenomena. Their advantages can be fully realized only if proper protection can be applied. The paper is devoted to a study of the protection requirements and the manner in which they depend on the basic properties of the rectifier itself. The protection requirements are

(*a*) Establish voltage and current ratings giving an acceptable life for the device.
(*b*) Control voltage and current surges generated by the equipment.
(*c*) Guard against occasional cell failures; devise means for detecting these and disconnecting faulty cells before cumulative damage occurs.
(*d*) Co-ordinate the protective measures to prevent unnecessary interruption of output.

To protect properly it is essential to know the margin between the safe working and failure limits for both current and voltage under both steady-state and transient conditions. It is also necessary to have some knowledge of the reasons for failure if the protection is to be better than a mere trial-and-error system. The rectifier properties are studied below, and suitable protection proposals are made on the basis of this study.

(2) VOLTAGE CHARACTERISTICS OF SEMICONDUCTOR RECTIFIERS

Fig. 1 shows idealized characteristics for semiconductor rectifiers. The forward voltage drop is small, substantially constant,

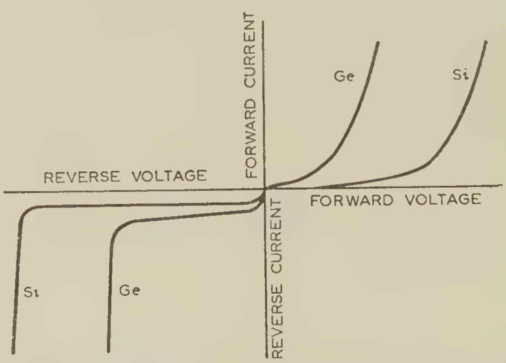

Fig. 1.—Idealized characteristics of typical high-power silicon and germanium diodes.

largely determined by the threshold voltage and of little consequence when designing protective systems. All rectifier cell failures eventually result in a failure to withstand the reverse voltage, even though this failure may have been caused initially by over-temperature or over-current. Cells are produced with a wide range of breakdown voltage; this must be determined under working conditions for each cell used in an equipment, and must be maintained throughout life.

(2.1) Reverse-Voltage Breakdown

The voltage rating of the cell is that reverse voltage which it can withstand successfully, and appears simple to determine from Fig. 1, which shows a very rapid rise of leakage current above a critical voltage (turnover voltage). If this voltage is exceeded, the reverse power loss becomes excessive and rapid breakdown follows.[7, 16] The total energy causing destruction in this way is

Mr. Corbyn and Mr. Potter are with the English Electric Co. Ltd.

much less than that required to raise the junction temperature dangerously high during forward conduction. The breakdown phenomenon is not simple, and is probably dependent upon local effects in the junction.

The reverse-voltage curve is best determined on load under working conditions, either real or simulated (a dynamic test). Both germanium and silicon rectifiers show decreased breakdown voltage as the junction temperature rises, and some rectifier cells show various other effects, including reverse ageing of junctions with time.[14] The detailed study of these effects does not fall within the scope of the paper, but stable cells are essential for a coherent protection policy.

(2.2) Measurement of Reverse Characteristics

Several methods are available for dynamic tests on rectifiers.

(*a*) *Electronic Methods using the Main Circuit Voltage.*[1]—The same voltage is used for producing forward current and measuring reverse voltage. This makes it difficult to trace the complete reverse characteristic if the rectifier cell is operated near the turnover voltage, since accidental breakdown can easily be caused.

(*b*) *Synthetic Voltage System.*—This requires two power supplies, one for reverse voltage and one for forward current. The disadvantage as a research tool is that exploration of the instantaneous reverse characteristic at different points of the cycle is not easily possible, since the same frequency of supply is used to produce both forward current and reverse voltage. The method is satisfactory for routine testing.

(*c*) *Synchronous Contactor Method.*[4]—This method is versatile, and in some variations the circuit can be opened at any selected point on the reverse half-cycle. In a refinement of this method a short h.f. pulse of variable amplitude is applied at a chosen point on each reverse half-cycle, and a complete characteristic is drawn up to turnover voltage. This has been found very satisfactory as a research tool, and has shown some unexpected effects not readily observed by other methods.

With germanium rectifiers the reverse saturation current is dependent on temperature at a low voltage. By applying a few volts to the junction during the reverse period with the synchronous contactor open, the junction temperature can be determined accurately by measuring leakage current with an oscillograph. Relationships between reverse leakage current and temperature are less simple with silicon rectifiers.

(2.3) Practical Voltage Ratings

To assign a voltage rating, it is essential to determine the limiting voltage for each cell by a routine test. The recurrent inverse voltage must be below the breakdown value. The exact ratio of recurrent peak inverse voltage to breakdown voltage can be chosen to suit different applications. The main sources of voltage surges are described below, and Fig. 11 shows the commoner methods of limitation.

(*a*) *Hole-Storage Voltages.*—The phenomenon of hole storage is described in Reference 12. The resulting voltages are reduced by means of small capacitors connected either across the a.c. input to the rectifier or across the output of the rectifier.[15] The latter solution is applicable only to bridge-connected full-wave rectifiers.

(*b*) *Transformer Switching-Surge Voltages.*—These are due to both electromagnetic and electrostatic coupling. The electromagnetic voltage surge caused by the changing flux in the transformer is usually greatest when switching out an unloaded transformer. The exact magnitude is uncertain and depends greatly on the switchgear. More than thrice normal is not common, but 10–15 times normal can occur in special circumstances. Suitably chosen capacitors and damping resistors in the input or output circuits of the rectifier or its transformer are very commonly applied to limit surge voltage.

Capacitive coupling between primary and secondary may cause the transformer secondary to rise above earth potential to almost 60% of the primary voltage to earth at the instant of switching in, irrespective of transformer voltage ratio. One cure is to connect a small capacitor of the low-inductance pattern between each output line and earth[15] (Fig. 11). An alternative protection is to fit an earth screen between the primary and secondary of the transformer, but this is not always practicable with large transformers.

(*c*) *Lightning Surges.*—These usually occur only on equipment with outdoor transformers fed by overhead lines. The surge voltage is commonly limited by surge diverters, usually connected from the main transformer primary to earth, but sometimes connected to the secondary. With modern diverters it is practicable to limit voltage surges and the residual voltage to as little as $1 \cdot 7$ times the normal recurrent peak voltage.

(*d*) *Operation of Protective Fuses.*—Rectifier fuses are usually designed to limit the arc voltage when clearing a fault to a value below the breakdown voltage of the rectifier cells (see Section 6.1).

(*e*) *Chopping of Load Current.*—This may be caused by the load itself or a circuit-breaker, and may result in severe voltage surges. Curative measures are discussed in Sections 9.1 and 9.2.

(3) CURRENT CHARACTERISTICS OF SEMICONDUCTOR RECTIFIERS

The junction temperature rise—an important factor in semiconductor rectifiers—is caused mainly by forward losses. In small rectifier cells the reverse losses may also be significant. The junction temperature must not cause any ageing effects during normal service, and must be sufficiently below any dangerous temperature to allow a margin for overloads or faults to be controlled by the protective gear without harm to the rectifier cells.

(3.1) Control of Junction Temperature

All semiconductor rectifiers have certain common features in their cooling systems, namely

(*a*) The rectifier consists of a thin wafer of the desired semiconductor material with one or more rectifying junctions.

(*b*) This wafer is soldered to heat sinks (usually copper blocks) above and below, using the thinnest possible solders to obtain low thermal resistance. Sometimes intermediate layers are added to reduce thermal stress.

(*c*) The heat sinks are thermally connected to cooling fins for transfer of losses to air, or to water boxes for transfer of the heat to water or other liquid.

The junction itself is of small thermal mass. Direct measurement of its temperature is impossible, and various indirect methods are used.

(3.2) Estimation of Junction Temperature

Fig. 2 shows a typical variation of junction temperature throughout the cycle. During the reverse voltage period it can be estimated as described in Section 2.2(*c*), and during the conduction period by calculation of heat-transfer rates,[2] although this is laborious and unsatisfactory. An alternative method[9] is to use an electrical analogue of the thermal system, and to feed into it a current waveform similar to that expected in service. The junction temperature is represented by the voltage shown on an oscillograph. Junction temperatures during the reverse-voltage period obtained by synchronous-contactor and thermal-analogue methods are in excellent agreement at normal loads.

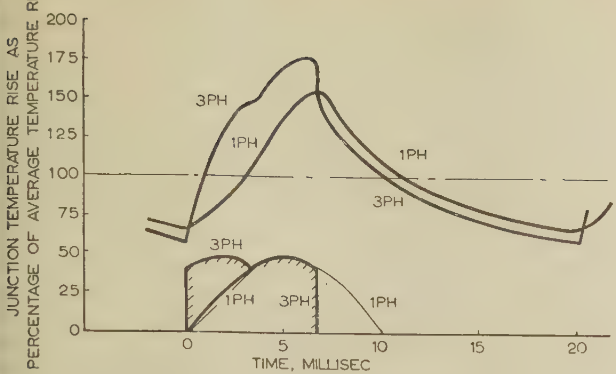

Fig. 2.—Temperature of a semiconductor junction above cooling medium during a complete cycle.

(3.3) Practical Current Ratings

A large heat sink for the wafer gives good cooling during overloads of a few seconds, with only a small effect on continuous rating. Good heat transfer to the ultimate sink of heat gives a high continuous rating with only a small effect on overload temperature rise.

Requirements of commercial ratings are

(*a*) Rated load current must not cause any significant deterioration of the rectifier within the life of the equipment. The maximum safe current may be limited by differential expansions of the various materials independently of actual junction temperature, so that there is a limit to the safe continuous current, however effective the cooling becomes.

(*b*) Rated overloads are the transient currents expected in commercial operation, and must not harm the rectifier junctions. During overloads the junction temperature rises, but a safety margin is essential, so that any fault applied during the overload will not cause a dangerous temperature rise before the protective gear operates.[16]

(*c*) Fault currents must be interrupted by the protective gear before any damage occurs to the equipment.

(3.3.1) The Survival Curve.

Fig. 3 shows an idealized survival curve which, for very short times, is a constant-coulomb line and is fixed by maximum junction temperature for long times. The exact shape is determined by the whole heat transfer system. Germanium junctions are destroyed at once when a temperature of 153°C is reached. This has been confirmed under a great variety of conditions, and

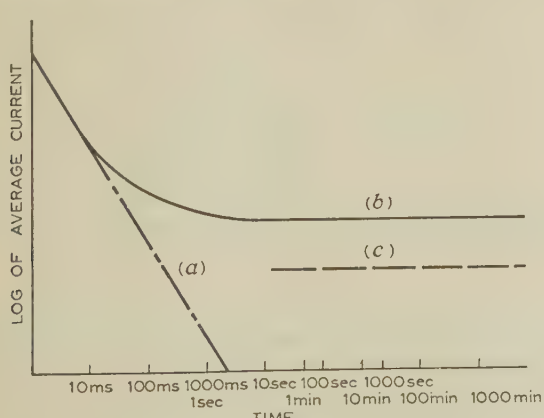

Fig. 3.—Theoretical type of survival curve for semiconductor rectifiers.

(*a*) Constant-coulomb line for failure in very short pulses.
(*b*) Complete failure.
(*c*) Normal full-load current.

is very close to the melting-point of indium. The constant-coulomb limit only applies for times of a few milliseconds. No sharp temperature limit is known for silicon rectifiers. Fig. 4 shows two practical survival curves for germanium cells, which converge after a few minutes even though the starting temperatures are different.

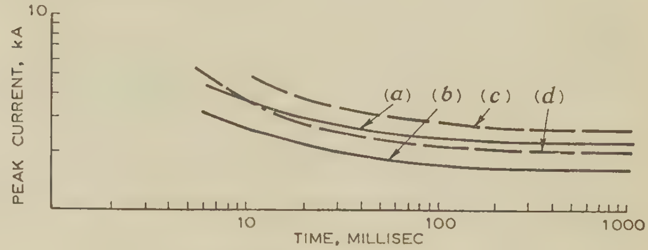

Fig. 4.—Typical rectifier and fuse survival curves.

(*a*) Fuse A.
(*b*) Fuse B.
(*c*) Cell failure curve (start 65° C peak junction—normal load).
(*d*) Cell failure curve (start 90° C peak junction—overload).

The rating and protection policy must ensure that the survival limits are never exceeded. This can be done most obviously by greatly increasing the reactance, but this solution is desirable in only a very few cases.

(4) NORMAL SERVICE CONDITIONS

The individual semiconductor cell is a low-power device, and large numbers are required in series and parallel for large powers. Half-wave circuits are usually employed where the maximum voltage of one cell permits, but for higher voltages bridge circuits, with their smaller and cheaper transformers, are almost universal.

(4.1) Series Connection of Rectifier Cells

Fig. 5(*a*) shows series strings connected in parallel in one arm of a rectifier. Some manufacturers ensure correct distribution

Fig. 5.—Series and parallel connections of rectifier cells.

of reverse voltage by connecting a small resistor across each cell; others use arrangements of transformers;[10] but it is often possible to grade cells so as to eliminate all aids to voltage sharing.[12]

(4.2) Parallel Connection of Rectifier Cells

For equipments employing one series rectifier cell per arm, or connected as Fig. 5(*b*), special care is necessary to ensure good current sharing between the parallel rectifier cells. The usual practice is to grade the cells.[14] Some writers recommend the universal use of current-balancing reactors.[11] It is essential to ensure good current sharing between cells on overload as well as on normal load.

Current sharing between the strings shown in Fig. 5(*a*) is good, since the characteristics average out.

(4.3) Series-Parallel Connection of Rectifier Cells

Fig. 5(*a*) and 5(*b*) show two possible connections. Those in Fig. 5(*a*) are generally cheaper, and current sharing between parallel strings requires small derating factors on current; those in Fig. 5(*b*) allow detection of a single cell failure without blowing

fuses, but an extra group of cells in series is required in each arm for continued operation with a faulty cell.

(5) FAULT CONDITIONS

Rectifier cells can fail either by over-voltage or over-current (see Sections 2 and 3). Typical survival curves are shown in Fig. 4.

External faults on an equipment can cause over-currents. Internal faults such as the backfire of a rectifier cell also cause over-currents in healthy cells. All external faults, if not dealt with correctly may cause breakdown of one or more rectifier cells, and turn the external fault into an internal fault. After a backfire the faulty cell is of no further value, but it is essential either to disconnect it or disconnect the whole equipment to prevent further damage.

(5.1) The Backfire Fault

When a rectifier cell fails to withstand the reverse voltage (i.e. backfires) at the end of its conduction period, it becomes short-circuited. Fig. 6 shows a 3-phase bridge circuit in which rectifier 1 has failed at the end of its normal conduction period at time t_1.

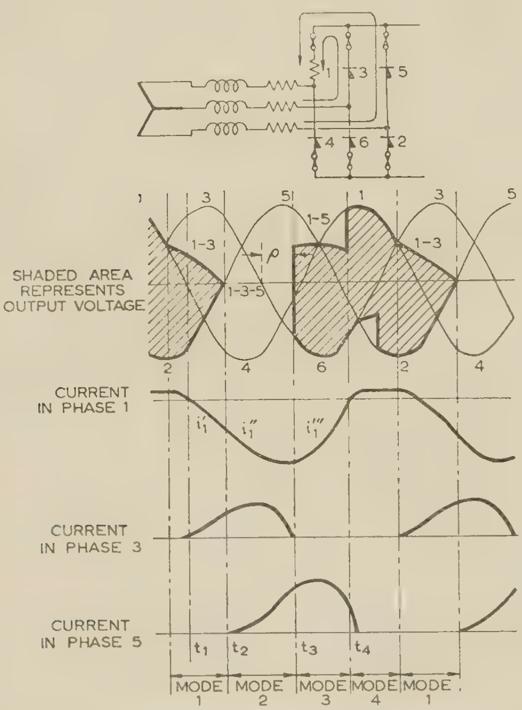

Fig. 6.—Backfire fault conditions in 3-phase bridge circuit.

This imposes a line-to-line fault on the transformer through rectifier 3 until time t_2. At this instant rectifier 5 will conduct, and a 3-phase fault occurs with zero output volts until time t_3, when the current in rectifier 3 becomes zero. From t_3 to t_4 the line-to-line fault involves rectifiers 1 and 5, with rectifier 3 withstanding an inverse voltage. From t_4 until the end of the cycle conditions are normal and the circuit shows no fault. The instant t_3 is determined by the resistance/reactance ratio of the circuit and the detailed equations are set out in Section 13.1. Fig. 7 gives a set of curves for calculation of backfire currents.

A rectifier cell failure during conduction still appears as a backfire fault at the end of conduction. These faults produce the highest peak currents, and require either that the busbars

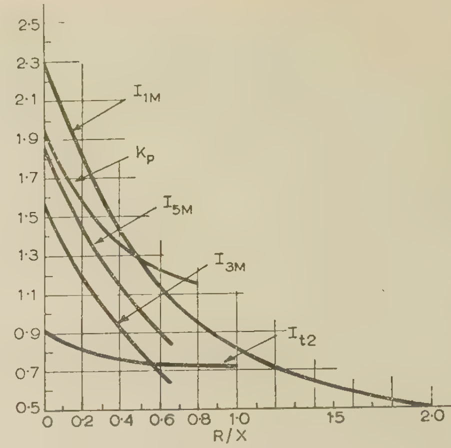

Fig. 7.—Curves for calculating backfire fault currents.

Base current for double star: $58I_d/X\%$.
Base current for 3-phase bridge: $116I_d/X\%$.
Actual current = I_{1M}, etc., times base current.
I_{1M} = Peak current in backfiring rectifier.
I_{3M} = Peak current in rectifier 3.
I_{5M} = Peak current in rectifier 5.
I_{t2} = Current in rectifiers 1 and 3 at t_2.
K_p = Asymmetry factor during backfire.
N.B. The curves give peak currents during the first cycle of fault current.

and transformer windings shall be mechanically strong enough to withstand the forces involved or that the protective device shall limit the fault current. Semiconductor rectifiers require fault removal in about 5 millisec to prevent the fault spreading. This ensures that peak prospective fault currents never flow in practice.

The 3-phase double-star circuit is similar in action to the bridge, except that when the load is regenerative the current is increased in the faulty rectifier and decreased in the rectifiers feeding into the fault.

For single-phase circuits, backfires behave as intermittent short-circuits between line terminals, and can be calculated by ordinary a.c. system theory.

(5.2) Effect of Series-Parallel Connection of Cells under Fault Conditions

(*a*) *One Rectifier Cell per Arm* (Fig. 6).—There are initially equal currents in rectifiers 1 and 3, and if individual fuses are used it is likely that two fuses will blow.

(*b*) *Many Rectifier Cells in Parallel per Arm* [Fig. 5(*a*)].—Backfire occurs when a complete string fails. The resultant current is concentrated in one string. The current fed in from the healthy rectifier arm is shared between many fuses and rectifier cells. Only one fuse blows. Action taken when a fuse blows will depend on the size of the equipment (Section 9).

(*c*) *Parallel Groups of Cells connected in Series* [Fig. 5(*b*)].—A failure of one cell to short-circuit produces no fault current, but redistributes the reverse voltage among the healthy cells. Continued operation is satisfactory provided that one extra group of cells in series has been provided in each arm.

(5.3) Calculation of Prospective Peak Backfire Currents

The currents in the circuit shown in Fig. 6 have been calculated for two cases as follow:

(*a*) Consider a 3-phase bridge of six rectifier cells, in which each rectifier cell and fuse has a resistance of 100 microhms.
Nominal output: 50 volts, 700 amp.
Transformer input: 38·5 kVA.
Transformer leakage reactance: 6%.
Transformer resistance: 3%.
Transformer secondary line voltage: 40·8 volts (23·5 volts/phase).

The base current of Fig. 7 is:

Steady-state peak output current on short-circuit: 13·5 kA.
Peak prospective current in phase 1: 15·9 kA (backfire).
Actual current in phase 1 after 5 millisec: 10 kA at t_2.

(*b*) Now consider ten such bridges in parallel, with a backfire of one cell. The nominal output is then 50 volts and 7 kA.

With the same resistance/reactance ratio and secondary voltage, the corresponding currents are

Steady-state peak output current on short-circuit: 135 kA.
Peak prospective current in phase 1: 115 kA (backfire).
Actual current in phase 1 after 5 millisec: 96 kA (backfire) at t_2.

The new prospective fault current is seven times, not ten times, the old value, since the fault is all concentrated in one fuse and string of cells. In practice, currents are further reduced, since account has not been taken of the extra resistance introduced by concentrating all the fault current on one rectifier cell and its associated busbars instead of ten in parallel.

(5.4) D.C. System Short-circuits

The backfire fault is easy to clear. The most critical fault is a short-circuit on the output just too small to operate the protection during the first conduction period. In Fig. 4, fuse B gives complete protection even to a d.c. short-circuit imposed during a heavy overload. Fuse A is adequate for backfire protection, but does not always protect against a d.c. short-circuit.

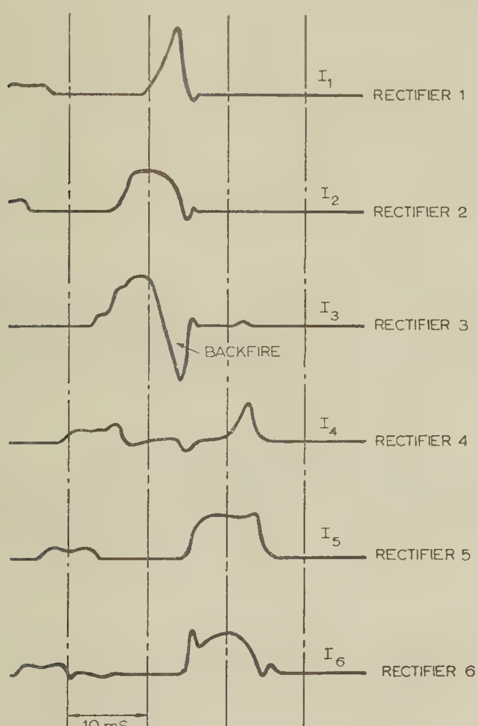

Fig. 8.—Current oscillograms on d.c. fault with one cell backfiring and a fuse giving only partial protection (see Section 5.4).

Fig. 8 shows the current when a d.c. fault was applied to the bridge in Fig. 6, using fuse A, when rectifier 3 was conducting. The junction temperature of rectifier 3 reached 153° C; the fuse did not clear before the end of conduction, the cell backfired when the reverse voltage was applied, and the fuse cleared during the reverse voltage period. Rectifier 5 received a full pulse of fault current, but of lower peak value than that of rectifier 3; its junction temperature was lower, the fuse cleared, no reverse voltage was applied and the cell survived. Fuse B is the correct type. Insufficient investigation could lead to the use of fuse A and a false sense of security.

Rectifier cells in parallel must share overload and fault currents correctly, but no other special precautions are necessary for series and parallel connections of many rectifier cells.

(5.5) D.C. System Overloads

Cell protective fuses may generally be permitted to blow under the rare conditions of a d.c. short-circuit, since a serious breakdown is thereby avoided. Fuse blowing must be prevented during normal overloads (see Section 8).

(6) PREDICTION OF PROTECTION REQUIREMENTS

Fuses are blown by the energy required to melt the silver elements, the energy being a function of the r.m.s. current.[17]

$$\text{Energy} = \int vi dt \text{ joules.}$$

In a resistive circuit $v = iR$

$$\text{and energy} = R \int i^2 dt = RI_{rms}^2 T \quad . \quad . \quad . \quad . \quad (1)$$

Heat is generated in a semiconductor rectifier during the reverse-voltage period by the leakage current. This is usually negligible compared with the forward loss. During forward conduction there is a voltage drop, which is about 0·6 volt for germanium and 1·2 volts for silicon.

$$\text{Energy loss in rectifier} = \int vi dt \text{ joules}$$

It is sufficiently accurate to assume the voltage to be constant, so that

$$\text{Energy loss} = v \int i dt = vQ \text{ joules} \quad . \quad . \quad . \quad (2)$$

Q is the quantity of electricity passed through the crystal. The temperature of the crystal is a function of Q and hence of the mean current.

For times less than 10 millisec the fuse and rectifier can be matched by comparison between the r.m.s. and mean values of the current through them.

The shape and magnitude of the pulse which the rectifier can withstand is determined experimentally, and a fuse is chosen which blows ahead of the rectifier (Fig. 4).

The relationship between the mean and r.m.s. values of current pulses is required. Under fault conditions the current pulses will usually be sinusoidal in form, with varying asymmetry. Section 13.2 gives the relations for unidirectional half-sine-wave pulses and fully displaced sinusoidal waveform as

$$\hat{I}_{rms} = \hat{I} \times K_r \text{ and } I_{mean} = \hat{I} \times K_M \quad . \quad . \quad . \quad (3)$$

$$\text{Hence} \qquad I_{rms} = \frac{Q}{T}\frac{K_r}{K_M} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

so that $\int i^2 dt$ can be determined.

Alternatively, the fuse cut-off current can be determined. In addition to the energy required to melt the fuse element, arc energy will be let through by the fuse, and a margin must be allowed for this.[17] Full-scale tests must be applied because of the complicated heat dissipation of rectifiers and fuses. Rectifier thermal characteristics can be studied with the analogue.[9] If the energy required to destroy the cell and the reverse-voltage failure limit depend on junction temperature, prediction of the fuse requirements becomes possible.

(6.1) Fuse Arc Voltages

Fig. 9 shows typical rectifier-cell voltages in a 3-phase bridge circuit when a fuse in one arm blows. Voltage spikes are produced. If the fuses are placed in the a.c. lines feeding a single bridge (Fig. 12, case 3), this effect is considerably reduced, but the current rating is $\sqrt{2}$ times that for fuses in the arms.

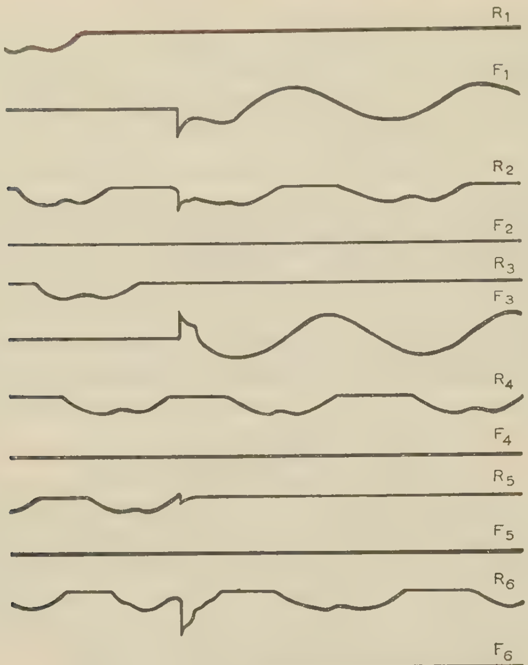

Fig. 9.—Voltages in a 3-phase bridge circuit during fuse blowing.

Where several bridges with their own line fuses are connected in parallel, fuse arc voltages appear across healthy rectifier cells as with other connections.

Fuses in series with rectifier cells (Fig. 12, case 1) remove the d.c. system short-circuit if rectifier cells in two opposite arms fail simultaneously. This failure is unlikely, but with many equipments in parallel or a load composed of motors or electrolytic cells with inherent back-e.m.f.'s, its importance cannot be ignored, since many semiconductor installations operate without d.c. circuit-breakers.

A satisfactory protective fuse must clear on fault without producing excessive over-voltages liable to damage healthy rectifier cells.

(7) H.R.C. FUSES FOR SEMICONDUCTOR PROTECTION

The foregoing discussion has shown that the special h.r.c. fuse satisfies most requirements for the protection of semiconductor rectifier cells.

(7.1) Fuse Requirements for Semiconductor Rectifiers

(*a*) High rupturing capacity is necessary to interrupt high fault currents (Section 5).
(*b*) Fuse must limit the total energy dissipated in the rectifier cell during a fault to a safe value (Section 3.3.1).
(*c*) Peak let-through current during a fault must be limited to a safe value (Figs. 4 and 10).
(*d*) Fuse arc voltage must not exceed the withstand limit of the rectifier cell (Section 6.1).
(*e*) The fuse must carry full load current indefinitely without deterioration.
(*f*) The fuse must be small for mounting near rectifier cells.
(*g*) Fuse characteristics must be capable of co-ordination with the time/current characteristics of other protective devices.

It is found that the industrial h.r.c. fuse does not generally satisfy these requirements, owing to its high arc voltage and excessive let-through current and energy.

In a few very small equipments industrial fuses can be placed in the input to a complete bridge where the fuse arc voltage is unimportant (see Section 6.1 and Fig. 12).

(7.2) H.R.C. Fuses for Semiconductor Rectifiers

Special fuses have been developed which satisfy the requirements of Section 7.1 using the essential features of the industrial h.r.c. fuse. The fuse protects the rectifier cell, has limited arc voltage and is cooled to give the highest continuous current rating. Table 1 gives some typical characteristics. For any rectifier cell, one fuse will just give protection, irrespective of cell loading. Fuses are not used as protection against small overloads. If cell rating increases, by improvement in cooling, there is little change in the peak currents causing failure, and the same fuse is used. Eventually the rated currents of rectifier and fuse approach one another, and cell overloads must be restricted because of the fuse. For these and other reasons there is a limit to the increase of rating obtainable by improved cooling.

(7.3) Current Limitation and Speed of Operation

In Section 6 it was shown that, if the survival limit of the semiconductor rectifier cell can be given in terms of the let-through energy and peak current, a fuse can be designed to give protection against faults. The provision of adequate rupturing capacity has not proved difficult in practice.

The peak let-through current is determined by the circuit and the pre-arcing characteristic of the fuse,[17] which is a function of fuse design. It is therefore possible to predetermine the maximum peak let-through current and to confirm this by test. The peak let-through current is adequately limited only if the rated

Table 1

CHARACTERISTICS OF TYPICAL FUSES FOR SEMICONDUCTOR RECTIFIERS

Fuse	Peak inverse voltage rating	Nominal r.m.s. current rating	Arc voltage		Let-through values at maximum applied voltage, and prospective current at 0·3 power factor lagging		
			Minimum value	At maximum rated voltage	Peak current	Total $\int i^2dt$	Total $\int idt$
	volts	amps	volts	volts	kA	amp²-sec × 10³	coulombs
A	150	200	80	225	4·5	100	20
B	150	400	80	225	9	300	43
C	300	200	160	450	8·75	170	40
D	300	400	160	450	14	700	80
E	450	200	320	675	7	200	36
F	450	400	320	675	13	700	70

The peak asymmetrical prospective currents are as follows:

A and B: 70 kA.
C and D: 150 kA.
E and F: 200 kA.

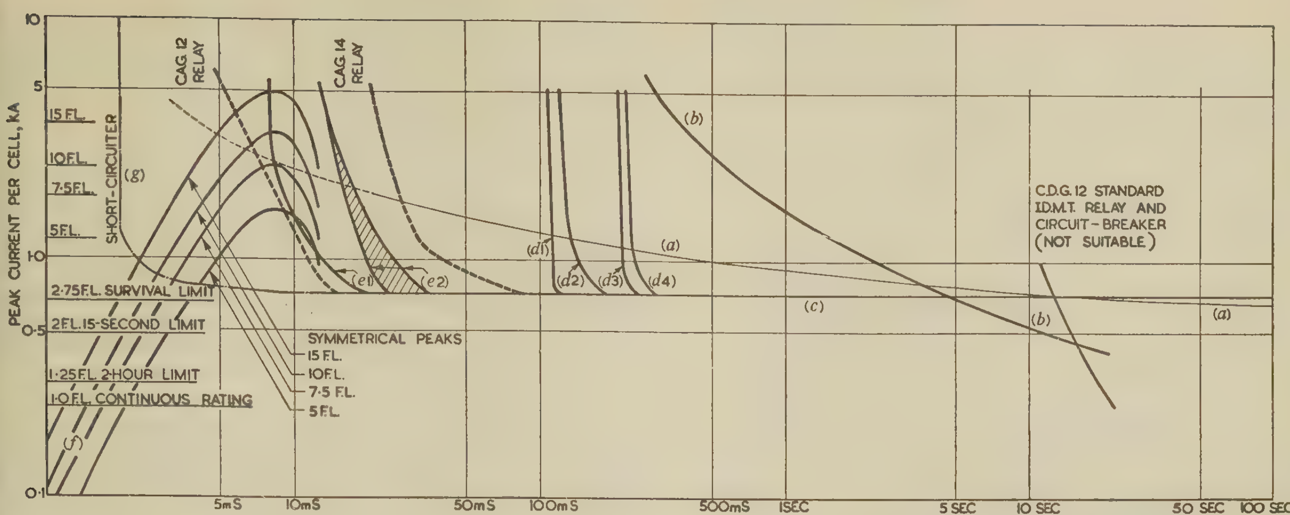

Fig. 10.—Co-ordination of complete over-current protection system for a typical semiconductor rectifier equipment.

The curves apply to the silicon cell of lowest quality acceptable for duty to B.S. 1698, class II.
(*a*) Survival curve for worst acceptable rectifier cell; the fuse survival curve should be just below this.
(*b*) I.D.M.T. relay (extremely inverse type CDG14) and circuit-breaker tripping curve.
(*c*) Minimum trip current on instantaneous relay (three times full load).
(*d*) Tripping characteristics of various relay and a.c. circuit-breaker combinations.
(*d*1) 5-cycle circuit-breaker and CAG12 fast relay.
(*d*2) 5-cycle circuit-breaker and CAG14 stabilized relay.
(*d*3) 10-cycle circuit breaker and CAG12 fast relay.
(*d*4) 10-cycle circuit breaker and CAG14 stabilized relay.
(*e*) Tripping characteristics of high-speed d.c. circuit-breaker
(*e*1) Direct trip.
(*e*2) With fast-relay trip (CAG12).
(*f*) First-cycle fault-current waves with maximum possible asymmetry for different supply reactances (drawn for $R/X = 0{\cdot}25$).
(*g*) Approximate tripping characteristic of short-circuiter.

fuse working voltage is not exceeded. If the semiconductor fuse is used in a circuit of too high a voltage, there is an increase of arcing time, consequent increasing It and a failure to protect the rectifier cell.

(8) CO-ORDINATION OF OVERALL PROTECTION

The preceding Sections have discussed the characteristics of semiconductor rectifiers, the hazards to which they are exposed and the means of protection. Overall protective requirements are examined below.

(8.1) Internal Faults

With series strings of cells [see Fig. 5(*a*)] one correctly chosen fuse per string gives complete protection. Failure of one string of cells blows this fuse, but there is no risk to fuses or cells in other arms of the rectifier, except in very small equipments containing only one string of cells per rectifier arm.

In Fig. 5(*b*) the failure of a rectifier cell-short-circuits the parallel group, and the reverse voltage is redistributed among the healthy groups. Individual rectifier cell fuses are useless, but a fuse can be either in the arm of the rectifier or on the input to the complete equipment to give overall protection. One extra group in series in each rectifier arm is required to prevent excessive reverse voltages on surviving cells following the failure of a single cell.

(8.2) External Faults

The guiding principle is that fuses blow only as a last resort, abnormal overloads and faults on the equipment being cleared by other means. The destruction of rectifier fuses simply because of a limited over-current must not occur.

Overall protection for all faults without unnecessary interruption of the load is achieved by co-ordinating the characteristics of the distribution system (whether this contains feeder circuit-breakers or fuses), the main output circuit-breaker (assuming one is used), the rectifier cell fuses, and the a.c. circuit-breaker and short-circuiter (if used). Fig. 10 gives a co-ordinated protection curve.

(8.3) Voltage Surges

The rectifier cell must be capable of withstanding a voltage above its normal recurrent peak inverse voltage, and the safety margin co-ordinated with surge prevention methods as discussed in Section 2.3. Fig. 11 shows some methods of surge-voltage prevention.

(9) PROTECTION IN PRACTICE

Fuses give satisfactory protection for semiconductor rectifier cells. Indicating striker-pin fuses are connected across the main fuse groups. When the main fuse clears, current is diverted through the small fuse, melts its element and a spring ejects a small striker pin. This pin operates a microswitch which can either give an alarm or trip the equipment. This system of warning is cheap.

It is not necessary to change fuses while the equipment is working, but rectifiers have been made with sections withdrawable under load, so that servicing can be carried out without interruption of output.

For small equipments the blowing of a fuse is made to trip the equipment. It is obvious that with only one, two or three parallel paths per rectifier arm, severe overloading of the remaining rectifier cells would occur if the equipment was not switched off. In large installations with many rectifier cells in parallel a warning of rectifier cell failure is adequate and allows the operator to replace the faulty cells when convenient. In borderline cases, methods are used by which the failure of one or more rectifier cells modifies the protective relay system and compels the operator to reduce load.

(9.1) General-Purpose Power Supplies

D.C. power supplies for factories, steelworks, docks, etc., usually have variable loads with frequent overloads. Main-

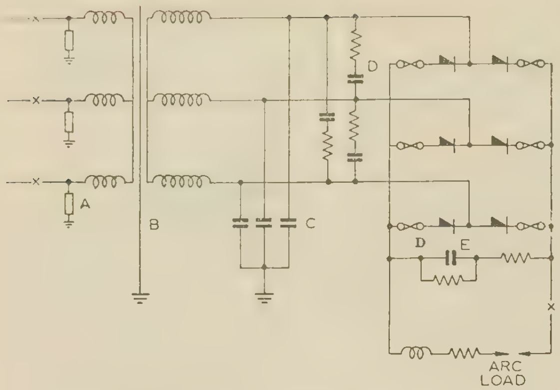

Fig. 11.—Methods of surge-voltage protection.

A Surge diverters for lightning protection.
B Earth screen. } Alternative protection against switching surges
C Low-inductance capacitors. } due to capacitive coupling between transformer windings.
D Capacitors for hole-storage and transformer-switching-surge protection.
E *RC* circuit to absorb magnetic energy (see Section 13.3).

tenance of supply is often vital, even though system short-circuits may occur.

In many systems the rectifier feeds the d.c. busbars through an isolator without any main d.c. circuit-breaker. The feeders would be supplied either through small feeder circuit-breakers or h.r.c. fuses. Where the percentage of total power on any one feeder is not too high, the risk of blown rectifier fuses is negligible, and the only case where this is likely to happen is with a direct short-circuit on the rectifier output. The equipment is constructed to render this unlikely. A small but important point is to see that busbars cannot be bridged by vermin.

It is usual to set the a.c. circuit-breaker instantaneous trip at 3½–4 times full load current, and use an i.d.m.t. relay to prevent long-term overloads. With these trip settings, measures may be necessary to prevent undesired trips by magnetizing inrush currents.

D.C. circuit-breakers are sometimes essential, and the risk of arc chopping with production of voltage surges has to be faced. To overcome this, a capacitor is connected across the output of the rectifier with resistance in series (Section 13.3.1 and Fig. 11). This is chosen to give any degree of voltage rise upon chopping the load current, and the resistance eliminates any voltage build-up due to resonances. For medium-voltage power supplies an electrolytic capacitor is satisfactory.

Some motors have regenerative braking, and when there is no other load on the system, a loading resistor switched in by over-voltage relays prevents dangerous voltage rises.[5]

For operation of semiconductor rectifiers in parallel with mercury-arc rectifiers, the semiconductor fuses may blow if a backfire occurs in the valve. Circuit-breakers are not fast enough to overcome this when low-reactance transformers are used, but satisfactory discrimination can be obtained if the mercury-arc rectifier is fitted with anode fuses.

If a semiconductor rectifier is operated in parallel with a mechanical rectifier, the short-circuiter gives such fast protection that the only solution is either to increase the reactance of the semiconductor equipment and then to rely on a circuit-breaker, or to use a short-circuiter on the semiconductor (Fig. 13).

(9.2) Furnace and Welding Loads

D.C. arc furnaces and welding equipment present special problems. Short-circuits are frequent, and the equipment must have a high reactance or be transductor (saturable reactor) controlled for operation down to short-circuit. Precautions are necessary in the design of transductor circuits to prevent severe surges of current or voltage.

Voltage rises due to chopping of the arc can be limited to a safe value by an *RC* circuit as shown in Fig. 11. The basis of calculation is given in Section 13.3.

(9.3) Electrolytic Loads

Loads are usually steady, and major short-circuits rare. D.C. circuit-breakers are often omitted, since rectifier cell fuses eliminate the possibility of a sustained short-circuit across the output busbars. Ordinary overloads can be cleared by the a.c. circuit-breakers. These installations are usually large, often supplied by e.h.v. overhead lines, and surge protection is necessary against damage due to lightning.

Industrial electroplating loads are similar in characteristic, but simpler, since only one equipment is involved and the lightning hazard does not usually exist. There is a risk of short-circuits on the output, but in practice these are of low current and hardly ever blow rectifier fuses.

(9.4) Alternator Excitation

These loads are free from voltage surges, and a short-circuit on the rectifier output is so rare that protection against it is not required. A shut-down is most undesirable, and rectifier sections are frequently made withdrawable under load for servicing.

With conventional alternators the rectifier can be connected direct to the field slip-rings without any circuit-breaker. It then provides a discharge path for the stored magnetic energy in the field, with almost its inherent time-constant. When the stator of an alternator is subjected to a short-circuit, a heavy 50 c/s current surge occurs in the excitation windings. The rectifier must be designed to withstand this surge without harm. These rectifiers can be supplied by an alternator exciter mounted on an extension of the main alternator shaft.

In the brushless alternator the rectifiers are mounted on the shaft. The exciter alternator has rotating a.c. windings and a stationary field system. Centrifugal accelerations may be large—up to 1 000g or over. The electrical requirements including those of reliability are the same as for the conventional alternator.

(9.5) Slip-Energy Recovery

This is a system in which a bridge-connected rectifier replaces the rotor resistance of a slip-ring motor. The rectifier output supplies the armature of a d.c. motor mounted on the same shaft as the rotor. The rectifier-motor combination appears as a variable resistance when seen from the rotor. Speed control is obtained, and the power absorbed from the rotor is returned to the shaft via the rectifier and motor armature. The risk of voltage and current surges is small. Precautions are taken to disconnect the rectifier should the rotor e.m.f. become too high through mishandling of the control gear.

(9.6) Traction

In some a.c. traction systems a high-voltage overhead line supplies transformers, rectifiers and control gear mounted on the locomotive or motor coaches. The semiconductor rectifier is steadily supplanting the mercury-arc rectifier for this duty. It is undesirable to use large numbers of fuses, and there is some preference for the cell connections in Fig. 5(*b*) with the number of fuses kept to two per single-phase bridge. Cell failures can be detected, and replacements carried out, in the running sheds. The rectifiers and circuit-breakers must be designed to withstand the over-currents resulting from motor flashovers without the blowing of any fuse.

The overhead lines are liable to voltage surges due to lightning, and the rectifiers must be designed with both sufficient voltage margin and surge-voltage limiting devices.

(9.7) Miscellaneous Loads

It is impossible to catalogue all the varied loads in detail, each of which has its own protective requirement.

Battery-charging installations tend to employ rectifiers with short-circuit current limited by ballast reactance. Only backfire protection need be considered. There is a small risk of a battery being connected with incorrect polarity, so that fuses in rectifier arms give satisfactory overall protection. Engine-starting, switch-closing and surge-absorption rectifiers must be designed to withstand heavy overload currents.

(10) CONCLUSIONS

The utilization of the new semiconductor rectifiers requires protection against over-currents and over-voltages. For protection against internal faults, such as the backfire of a rectifier cell, or ultimate protection in case of a direct short-circuit on the equipment output, the h.r.c. current-limiting fuse offers a satisfactory solution. A protective system using fuses can be devised for most applications still allowing the use of normal reactance in the transformers. By the proper placing of fuses in the circuits, d.c. circuit-breakers can often be eliminated. The clearing times of all devices needs careful co-ordination, as shown in Fig. 10.

For over-voltage protection it is essential to protect against hole-storage voltages, transformer switching surges and in some cases chopping of the load current either by d.c. circuit-breakers or by special loads. For installations fed by overhead lines it is essential to protect against lightning by means of surge diverters, and to provide an adequate voltage margin in the rectifier cells. When all the protective systems are co-ordinated excellent continuity of power supply can be assured, and at the same time the protective system need not impose any serious limitation on the use of semiconductor rectifiers.

The nomogram, Fig. 19, gives values of C and R for the surge-suppression circuit shown in Fig. 18; values are discussed in Section 13.3.

(11) ACKNOWLEDGMENTS

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(13) APPENDICES

List of Symbols

C = Capacitance.
i = Instantaneous current (subscript denotes phase number).

$\hat{I}$ = Peak current.
I_s = Secondary current (r.m.s.).
I_{mean} = Mean current.
I_{rms} = R.M.S. current.
I_d = Output current.
K_M = Ratio between mean and peak current.
K_r = Ratio between r.m.s. and peak currents.
K = Ratio between normal voltage and permissible rise in voltage.
L = Inductance.
M = Percentage magnetizing current at full load.
mI_s = Magnetizing current referred to secondary.
p = Fractional load.
R = Resistance.
R_d = Equivalent load resistance.
t = Time.
v = Instantaneous voltage.
V_s = Secondary r.m.s. voltage.
V_d = Output direct voltage.
$\omega = 2\pi f$ = Angular frequency.
$\phi = \arctan \omega L/R$.

Other symbols are explained in the text, as they arise.

(13.1) Backfire Fault Currents in a 3-Phase Bridge (Fig. 6)

(13.1.1) First Mode: 2-Phase Fault.

Consider backfire commencing at the instant t_0 when $v_1 = v_3$. During overlapping and subsequently,

$$\left.\begin{aligned} v_1 &= (\sqrt{2})V\cos\left(\omega t + \frac{\pi}{3}\right) \\ v_2 &= (\sqrt{2})V\cos\left(\omega t - \frac{\pi}{3}\right) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (5)$$

Neglecting voltage drops, the voltage between neutral point of the transformer and the d.c. busbar will be:

$$v_d = \frac{v_3 + v_1}{2} = \frac{\sqrt{2}}{2}V\cos\omega t \quad . \quad . \quad . \quad . \quad (6)$$

The equation for the circuit is

$$L\frac{di_1'}{dt} + Ri_1' = v_d - v_1 = (\sqrt{2})V\sin\frac{\pi}{3}\sin\omega t \quad . \quad . \quad (7)$$

which gives ($t = 0$ when $i_1' = I_D$)

$$i_1' = \frac{\sqrt{6}}{2}\frac{V}{Z}[\sin(\omega t - \phi) + \sin\phi\varepsilon^{-Rt/L}] - I_D\varepsilon^{-Rt/L} \quad . \quad (8)$$

The last factor is generally small compared with the first, and can be ignored. The current in arm 3 is equal to that in arm 1, so that

$$-i_3' = i_1' = \frac{\sqrt{6}}{2}\frac{V}{Z}[\sin(\omega t - \phi) + \sin\phi\varepsilon^{-Rt/L}] \quad . \quad (9)$$

This condition exists until rectifier 5 starts to conduct, i.e. when $\omega t = \pi/2$.

(13.1.2) Second Mode: 3-Phase Fault.

It is convenient to redefine the zero time for this mode. The instantaneous phase voltages are

$$\left.\begin{aligned} v_1 &= (\sqrt{2})V\cos(\omega t + 5\pi/6) \\ v_3 &= (\sqrt{2})V\cos(\omega t + \pi/6) \\ v_5 &= (\sqrt{2})V\cos(\omega t - \pi/2) \end{aligned}\right\} \quad . \quad . \quad . \quad (10)$$

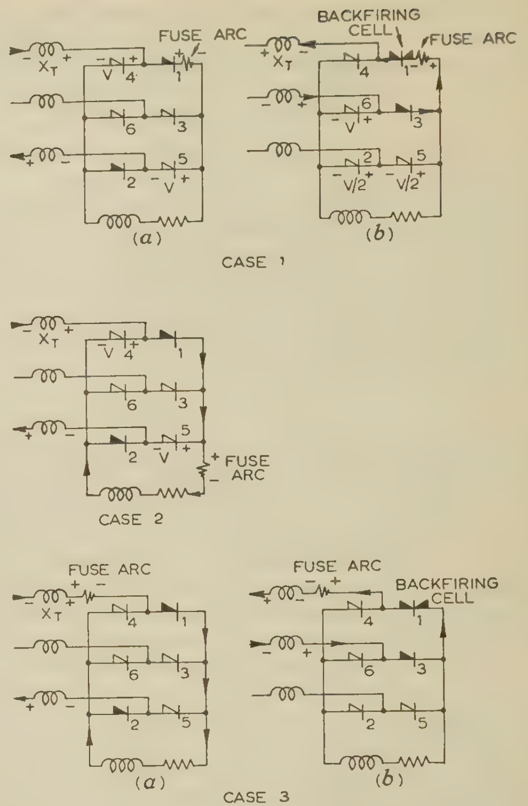

Fig. 12.—Location of fuse arc voltages in a 3-phase bridge.

Case 1: Fuses in rectifier arm (complete protection).
(a) Fuse clears on over-current.
(b) Fuse clears on backfire.
Case 2: Fuse or circuit-breaker in d.c. output.
Protection only against d.c. over-current.
Case 3: Fuse or circuit-breaker in a.c. input.
Complete protection but discrimination worse than case 1.
(a) Fuse clears on over-current.
(b) Fuse clears on backfire.
Rectifiers shown solid are those carrying current.
V indicates cells subject to severe voltage stress.

The current in phase 1 must now satisfy

$$L\frac{di_1''}{dt} + Ri_1'' = -(\sqrt{2})V\cos(\omega t + 5\pi/6)$$

which gives

$$i_1'' = -\frac{\sqrt{2}}{Z}V[\cos(\omega t + 5\pi/6 - \phi) + \cos(5\pi/6 - \phi)\varepsilon^{-Rt/L}] + i_1'\varepsilon^{-Rt/L} \quad . \quad (11)$$

where i_1' is given by eqn. (8) with $t = \pi/2\omega$. Similarly,

$$i_3'' = \frac{(\sqrt{2})V}{Z}[\cos(\omega t + \pi/6 - \phi) - \cos(\pi/6 - \phi)\varepsilon^{-Rt/L}] + i_3'\varepsilon^{-Rt/L} \quad . \quad (12)$$

and

$$i_5'' = \frac{(\sqrt{2})V}{Z}[\cos(\omega t - \pi/2 - \phi) - \sin\phi\varepsilon^{-Rt/L}] \quad . \quad (13)$$

This mode of operation is terminated when the current in phase 3 falls to zero. The most convenient way to determine this time is to insert the circuit constants into the equations for increasing values of time. It is a laborious process but can be quickly performed by digital computers.

(13.1.3) Third Mode: 2-Phase Short-Circuit.

When the current in phase 3 reaches zero it cannot reverse owing to the presence of a healthy rectifier in that phase. The

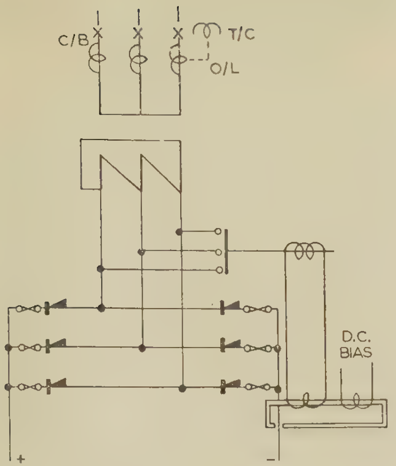

Fig. 13.—Typical connection of a short-circuiter.

ult therefore reverts to a 2-phase short-circuit in phases 1 and 5. he time of commencement of mode 3 is t_3, which is ρ seconds ter v_3 passes through zero (see Fig. 6). Hence it can be shown at

$$= \sqrt{\frac{3}{2}}\,\frac{V}{Z}[\cos(\omega t + \rho - \phi) - \cos(\rho - \phi)\varepsilon^{-Rt/L}] + i_1''\varepsilon^{-Rt/L} \quad . \quad . \quad . \quad . \quad (14)$$

here i_1''' is the final value of current i_1'' in mode 2. Since two hases only are involved, $i_5''' = -i_1'''$. This third mode terinates when i_5''' falls to zero. At this instant the voltage in hase 1 is positive, which corresponds to its normal conducting ne.

3.1.4) Fourth Mode: Normal Operation Resumed.

The normal direct output voltage therefore appears, and hase 1 will pass current through the load until the voltage of hase 3 again exceeds the voltage of phase 1. The cycle of vents already described is then repeated in the second and ubsequent cycles.

(13.2) Mean and R.M.S. Values for Periodic Waves

3.2.1) Mean Values of Sinusoidal Waves.

The general equation to a sine wave of current, including the witch-in transient, is

$$i = \hat{I}[\sin(\omega t - \phi) + \sin\phi\,\varepsilon^{-Rt/L}] \quad . \quad . \quad . \quad (15)$$

Therefore, for t seconds,

$$I_{mean} = \frac{\hat{I}}{t}\int_0^t [\sin(\omega t - \phi) + \sin\phi\,\varepsilon^{-Rt/L}]dt$$

$$= \frac{\hat{I}}{\omega t}\left\{[\cos\phi - \cos(\omega t - \phi)] + \frac{\omega L}{R}\sin\phi(1 - \varepsilon^{-Rt/L})\right\}$$

If $\phi = 0$, $\cos\phi = 1$, $\omega t = 0$, i.e. the fully symmetrical sine vave, then

$$I_{mean} = \frac{\hat{I}}{\omega t}(1 - \cos\omega t) \quad . \quad . \quad . \quad . \quad (16)$$

nd if $\phi = \pi/2$, $\cos\phi = 0$, $R = 0$, i.e. fully displaced sine wave, hen

$$I_{mean} = \hat{I}\left[1 - \frac{\cos(\omega t - \pi/2)}{\omega t}\right] \quad . \quad . \quad . \quad (17)$$

In eqns. (16) and (17) $I_{mean} = \hat{I}K_M$. Therefore

$$K_M = \frac{1}{\omega t}(1 - \cos\omega t) \quad . \quad . \quad . \quad . \quad (18)$$

for a symmetrical half-sine-wave pulse lasting π radians

and $$K_M = \left[1 - \frac{\cos(\omega t - \pi/2)}{\omega t}\right] \quad . \quad . \quad . \quad (19)$$

for a fully displaced symmetrical wave.

The rectifier cells and fuses conduct only half sine waves, and the value of K_M for such times of pulses is determined as follows:

The value of K_M at the end of the first pulse is

$$K_{M1} = \frac{1}{\omega t_1}(1 - \cos\omega t_1) = \frac{2}{\pi}$$

and during the succeeding half-cycle it falls to

$$K_{M2} = K_{M1}\frac{t_1}{t_2}$$

During the next half-cycle current is conducted, so that the mean value at the end of the pulse is proportional to K_{M3}, where

$$K_{M3} = K_{M2}\frac{t_2}{t_3} + K_{M1}\left(\frac{t_3 - t_2}{t_3}\right)$$

and $$K_{M1} = \frac{1}{\omega(t_3 - t_2)}[1 - \cos\omega(t_3 - t_2)]$$

The general form is then

$$K_{Mn} = K_{Mn-1}\left(\frac{t_{n-1}}{t_n}\right) + K_M\left(\frac{t_n - t_{n-1}}{t_n}\right) \quad . \quad . \quad (20)$$

for odd values of n,

and $$K_{Mn} = K_{Mn-1}\left(\frac{t_{n-1}}{t_n}\right) \quad . \quad . \quad . \quad . \quad (21)$$

for even values of n.

By using these identities the values of K_M/t have been determined and are plotted for symmetrical sine waves in Figs. 14 and 15 and for fully displaced sine waves in Fig. 16.

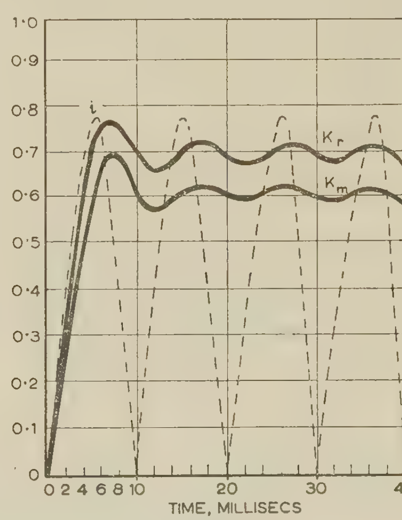

Fig. 14

i = Rectifier current plotted to an arbitrary scale.

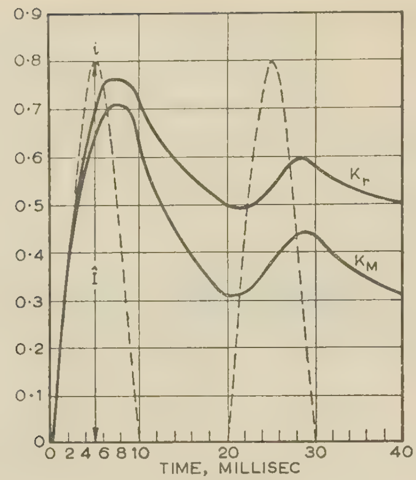

Fig. 15

= Rectifier current plotted to an arbitrary scale.

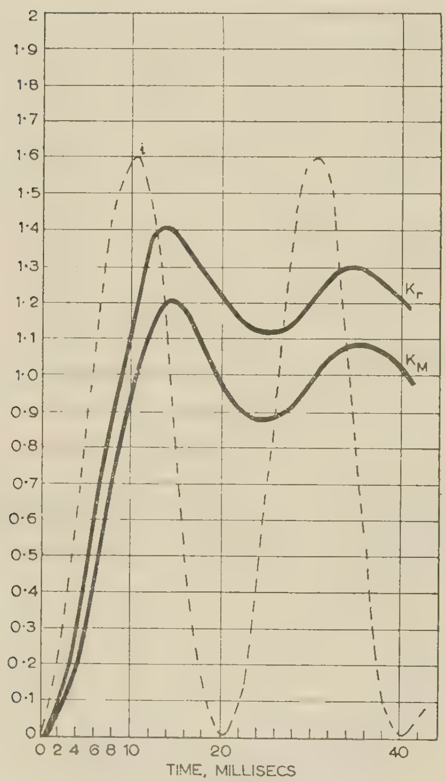

Fig. 16

i = Rectifier current plotted to an arbitrary scale.

(13.2.2) R.M.S. Values of Sinusoidal Waves.

The r.m.s. value of the general sine wave of current is given by

$$I_{rms} = \left\{\frac{\hat{I}^2}{t}\int_0^t [\sin(\omega t - \phi) + \sin\phi\varepsilon^{-Rt/L}]^2 dt\right\}^{1/2}$$

$$= \hat{I}\left[\frac{1}{2} - \frac{\sin 2\phi}{4\omega t} - \frac{\sin 2(\omega t - \phi)}{4\omega t} - \frac{L}{2Rt}\sin^2\phi(1 - \varepsilon^{-Rt/L}) - \frac{2\sin^2\phi\varepsilon^{-Rt/L}\sin\omega t}{\omega t}\right]^{1/2} \quad . \quad (22)$$

If $\phi = 0$, $\cos\phi = 1$, $\omega = 0$, i.e. fully symmetrical sine wave,

$$I_{rms} = \hat{I}\left(\frac{1}{2} - \frac{\sin 2\omega t}{4\omega t}\right)^{1/2} \quad . \quad . \quad . \quad . \quad (23)$$

and if $\phi = \pi/2$, $\cos\phi = 0$, $R = 0$, i.e. fully displaced sine wave,

$$I_{rms} = \hat{I}\left[\frac{3}{2} - \frac{\sin 2(\omega t - \pi/2)}{4\omega t} - \frac{\sin\omega t}{\omega t}\right]^{1/2} \quad . \quad (24)$$

In eqns. (23) and (24), $I_{rms} = \hat{I}K_r$. Therefore, for symmetrical sine waves,

$$K_r = \left(\frac{1}{2} - \frac{\sin 2\omega t}{4\omega t}\right)^{1/2} \quad . \quad . \quad . \quad . \quad (25)$$

and for fully displaced sine waves,

$$K_r = \left[\frac{3}{2} - \frac{\sin 2(\omega t - \pi/2)}{4\omega t} - \frac{\sin\omega t}{\omega t}\right]^{1/2} \quad . \quad . \quad (26)$$

These values can be treated in the same way as K_M for trains of waves.

The values of K_r have been calculated and are plotted in Figs. 14, 15 and 16.

(13.2.3) Mean Values of Square-Wave Pulses.

The mean value of square wave pulses is given by

$$I_{mean} = \hat{I}\frac{t_c}{T}$$

Where t_c = Conduction time.
T = Total time.
$\hat{I}$ = Peak value of current pulse.

i.e.
$$K_M = t_c/T \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

The variation of K_M with time is plotted in Fig. 17 for cyclic square-wave pulses of 120° duration.

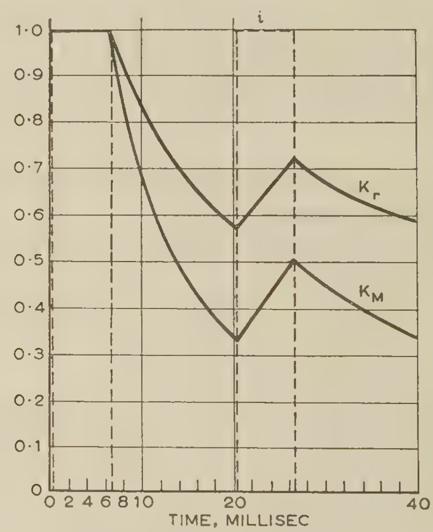

Fig. 17

i = Rectifier current plotted to an arbitrary scale.

(13.2.4) R.M.S. Values of Square-Wave Pulses.

$$I_{rms} = \hat{I}\sqrt{\frac{t_c}{T}}$$

Therefore
$$K_r = \sqrt{\frac{t_c}{T}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

The variation of K_r with time is plotted in Fig. 17 for cyclic square-wave pulses of 120° duration.

(13.2.5) Relationship between Quantity of Electricity and R.M.S. Current.

$$Q = I_{mean}T = \hat{I}K_M T$$

$$I_{rms} = K_r\hat{I}$$

$$\text{so that } Q = I_{rms}\frac{K_M}{K_r}T \quad . \quad . \quad . \quad . \quad (29)$$

(13.3) Control of Voltage Surges

Dangerous voltage rises on any rectifier equipment can be caused by (*a*) chopping of the load current, (*b*) switching out the transformer with rectifier on open-circuit, (*c*) and switching in the transformer (especially on an unloaded rectifier). A capacitor and resistor in series connected across the rectifier output will prevent these voltage rises becoming dangerous. The requirements are examined below for a bridge-connected rectifier (see Fig. 18). Assume 'rectangular' currents in the rectifier. The capacitor will be charged initially to V_d volts: allow it to charge to kV_d volts.

Regulation and the peak/mean voltage ratio at the output have been neglected, since the two effects are almost self-cancelling in practice.

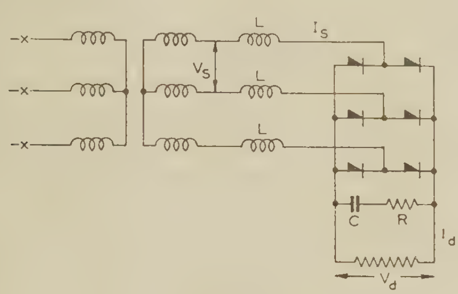

Fig. 18

(13.3.1) Voltage Rises due to Load-Current Chopping.

Assume that a rectifier equipment is delivering power when the load is suddenly chopped. The energy stored in the transformer leakage field will be transferred to any electric field present, and may cause a dangerous voltage rise if the capacitance is too small.

Energy stored per line at peak current $= \frac{1}{2}Lp^2I_d^2$ joules.
Initial energy in capacitor $= \frac{1}{2}CV_d^2$ joules.
Final energy in capacitor $= \frac{1}{2}CK^2V_d^2$ joules.

The worst case is if the current is chopped when two lines are carrying peak current.

Then, neglecting resistance loss,

$$2[\tfrac{1}{2}Lp^2I_d^2] = \tfrac{1}{2}CV_d^2(K^2 - 1)$$

$$\text{Therefore} \quad C = \frac{2L}{(K^2-1)}\left(\frac{pI_d}{V_d}\right)^2 = \frac{2L}{(K^2-1)}\frac{p^2}{R_d^2} \quad . \quad . \quad (30)$$

Now, in a 3-phase bridge, if X is the percentage reactance at full load,

$$V_d = 1{\cdot}35V_s,\ I_s = I_d\frac{\sqrt{2}}{\sqrt{3}} \text{ and } X = \frac{I_s\omega L}{V_s/\sqrt{3}} 100 \text{ per cent}$$

Where X, I_d and V_d refer to rated full load when $p = 1$.

$$\text{Hence} \quad L = \frac{X}{100(\sqrt{3})\omega}\frac{V_s}{I_s} = \frac{X}{191\omega}\frac{V_d}{I_d} \text{ or } \frac{XR_d}{191\omega}$$

$$\text{Therefore} \quad C = \frac{2}{191(K^2-1)}\frac{X}{\omega}\frac{p^2}{R_d} \quad . \quad . \quad . \quad . \quad (31)$$

i.e. the value of C depends only on load resistance R_d for a given reactance and frequency.

$$\text{In terms of power} \quad P = \frac{V_d^2}{R_d}$$

$$C = \frac{2}{191(K^2-1)}\frac{X}{\omega}p^2\frac{P}{V_d^2} \quad . \quad . \quad . \quad . \quad (32)$$

Breaking full-load current ($p = 1$) and allowing the capacitor to charge to $2V_d$ ($K = 2$) with $x = 5\%$ at full load gives values of C as follows:

Voltage	Capacitance					
	at 0·25 kA	at 0·5 kA	at 1 kA	at 2 kA	at 4 kA	at 8 kA
	µF	µF	µF	µF	µF	µF
63	223	445	890	1 980	3 960	7 920
125	112	225	449	898	1 790	3 592
250	56·1	112	224	448	896	1792
500	28	56·1	112	224	448	896
750	21	42	84	168	336	672
100	14	28	56·1	112	224	448

Since $C \propto p^2$ there is a drop in capacitor size for chopping at small currents. Also, $C \propto 1/(K^2 - 1)$, i.e. the capacitor size increases if the voltage rise is severely limited.

K	1 (no rise)	1·25	1·5	2·0	2·5	3·0
Constant $[= 1/(K^2 - 1)]$	∞	1·78	0·8	0·33	0·19	0·125
Relative capacitance . .	∞	5·5	2·42	1·0	0·575	0·3

(13.3.2) Switching out a Transformer with an Unloaded Rectifier.

If the transformer is excited normally, the magnetizing current per phase is mI_s amperes at $V_s/\sqrt{3}$ volts referred to the secondary.

Hence, if this current is deemed to be sinusoidal,

$$X_M = \frac{V_s}{mI_s\sqrt{3}} \text{ ohms and } L_M = \frac{V_s}{\omega mI_s\sqrt{3}} \text{ henrys}$$

The maximum energy stored per phase is approximately

$$\tfrac{1}{2}L_M(mI_s\sqrt{2})^2 = L_M m^2 I_s^2$$

so that the total energy is $1\frac{1}{2}L_M m^2 I_s^2$.

Assume that the magnetizing current is chopped infinitely suddenly and that all the energy is used to charge the capacitor from V_d volts to KV_d volts.

$$\text{Then} \quad 1\tfrac{1}{2}L_M m^2 I_s^2 = \tfrac{1}{2}CV_d^2(K^2 - 1)$$

Therefore

$$C = \frac{3L_m}{(K^2-1)}\frac{(mI_s)^2}{V_d^2} \text{ or } \frac{2L_m}{(K^2-1)}\frac{(mI_d)^2}{V_d^2} \text{ since } I_s = I_d\frac{\sqrt{2}}{\sqrt{3}} \ . \ (33)$$

This is exactly comparable to the 'leakage reactance' formula with L_M in place of L.

Conversion to terms of V_d and I_d gives

$$C = \frac{2}{1{\cdot}91(K^2-1)}\frac{m}{\omega}\frac{I_d}{V_d}$$

$$= \frac{2}{191(K^2-1)}\frac{M}{\omega}\frac{I_d}{V_d} \text{ farads} \quad . \quad . \quad . \quad (34)$$

This is comparable with the leakage reactance case where M, the percentage magnetizing current at full load, occurs instead of X, the percentage leakage reactance at full load.

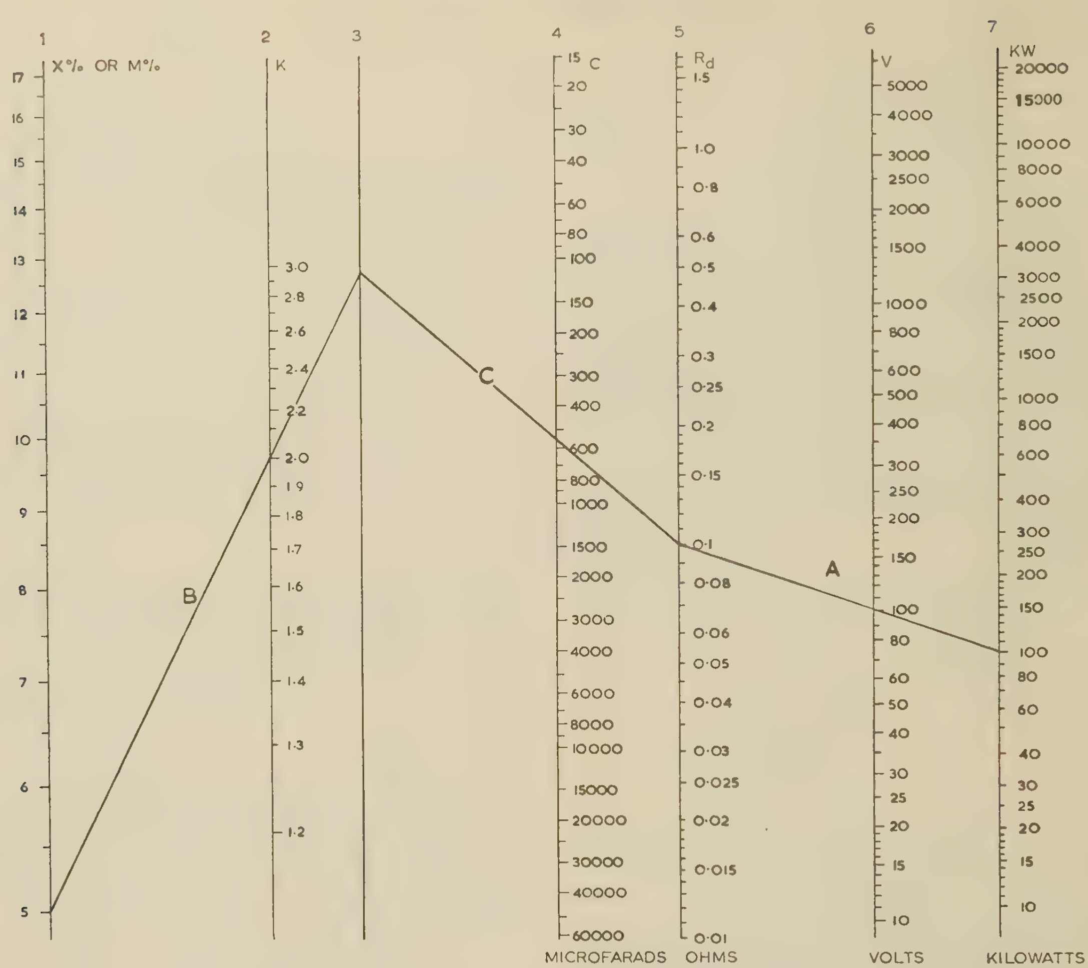

Fig. 19.—Nomogram for surge-voltage suppression circuit.

Draw line A through power and voltage values of the equipment on scales 6 and 7, thus obtaining the value of R_d on scale 5. Draw line B through leakage reactance value on scale 1, and the required value of K on scale 2 to the reference line 3. Then draw line C from the point on line 3 to the value of R_d on scale 5. This line then gives the value of C on scale 4. This is the value of C for $p = 1$; if $p \neq 1$ the value obtained must be multiplied by p^2.

(13.3.3) Prevention of Resonance and Shock Excitation.

Resonance occurs at an angular frequency ω_0, when

$$2\omega_0 L = \frac{1}{\omega_0 C} \text{ i.e. } \omega_0^2 = \frac{1}{2LC} \quad . \quad . \quad . \quad . \quad (35)$$

This assumes that the resonating circuit is composed of the leakage reactance in two phases together with the output capacitance.

From eqn. (30),

$$\left.\begin{aligned} LC &= \frac{2L^2}{(K^2-1)}\left(\frac{pI_d}{V_d}\right)^2 \\ \text{or} \quad LC &= \frac{C^2(K^2-1)}{2}\left(\frac{V_d}{pI_d}\right)^2 \end{aligned}\right\} \quad . \quad . \quad . \quad (36)$$

To prevent resonance building up, make $R = 2\omega_0 L = 1/\omega_0 C$, i.e. circuit Q-factor $Q_0 = 1$.

Then from eqns. (35) and (36),

$$R = \frac{R_d}{p}[K^2 - 1]^{1/2} \quad . \quad . \quad . \quad . \quad . \quad (37)$$

Now consider the condition for shock excitation.

The limiting condition is that $\left(\frac{R}{2L}\right)^2 \geqslant \frac{1}{LC}$

Therefore $$R = \left(\frac{4L^2}{LC}\right)^{1/2}$$

i.e. for critical damping

$$R = \frac{R_d}{p}[2(K^2-1)]^{1/2} \quad . \quad . \quad . \quad . \quad (38)$$

But, for minimum RC voltage when switching in,

$$R = \frac{R_d}{p} \times 0{\cdot}4(K^2-1)^{1/2} \quad . \quad . \quad . \quad . \quad (39)$$

Fig. 19 shows a nomogram calibrated in terms of R_d for calculating the values of R and C deduced in this Appendix.

(13.3.4) Permissible Values of R and C.

In practice, R can be less than the 'resonance' value, since some overshoot of capacitor voltage is permissible. Imagine chopping full-load current with $R = R_d$. The initial inrush current, I_d, to the capacitor then produces a voltage drop RI_d in the resistor equal to the full-load voltage, so that the initial voltage across the rectifier is twice normal.

Generally, if the inrush current is pI_d and the capacitor rises to KV_d volts, the permitted value of R is given by

$$pI_dR \leqslant V_d(K-1)$$

giving $$R \leqslant R_d\frac{(K-1)}{p} \text{ since } \frac{V_d}{I_d} = R_d \quad . \quad . \quad . \quad (40)$$

The resistance must be large enough to carry the capacitive ripple current continuously.

Of the four expressions for R, critical damping is always greatest, and above $K = \sqrt{2}$ the condition for minimum switch-in voltage gives the least value. Below $K = \sqrt{2}$ the condenser inrush value is least. The harmonic voltages are small, in a free-firing rectifier, and some small build-up of voltage at resonance is permissible; also some slight over-swing of voltage on the RC circuit is permissible when switching in. The most satisfactory compromise for general use is approximately $R = (R_d/p)(K-1)$.

The capacitances are based on the assumption of load or magnetizing currents chopped infinitely suddenly at peak values. In practice, smaller capacitances are usually adequate—often much smaller when only magnetizing-current chopping must be considered. It is unusual for d.c. circuit-breakers to chop currents above 100 amp, so that p is often quite small.

DISCUSSION BEFORE THE UTILIZATION SECTION, 10TH DECEMBER, 1959

Mr. J. A. Broughall: Railways need equipment which is completely reliable and as simple as possible, and unless great care is taken, the semiconductor rectifier—which has excellent potentialities—may become too complicated. It is vital that full use is made of the recent tremendous progress in the manufacture of these rectifiers, whereby the purity of the metals used and the method by which they are joined together are not 2–3 times, but 1 000 times, better than they were a few years ago. This should permit the elimination of protective devices, so that we may regard this rectifier exactly as we regard a switch: a switch gives protection, but needs nothing to protect it. Similarly, these rectifiers should need no protection after selection as the right type and size for the job. I am, of course, exaggerating a little in order to make my point, for traction applications demand some method of getting the engine away in the event of failure of one component; but it would be a great mistake to fit surge diverters and other forms of protection on a traction rectifier. One of the major advantages of the semiconductor rectifier is that it needs neither preheating nor ignition circuits; having eliminated these, we do not want to substitute other devices.

If this concept of rectifier usage is to succeed the user must state definitely the conditions with which the rectifier must comply in terms of the parameters which are important to rectifier designers. Since these rectifiers have very short time-constants and therefore no overload capacity, they form the weakest link in the chain—rectifier, transformer and motor. The whole installation must be dealt with as a unit, with both sides knowing what they are doing. The user must determine the operating conditions for the rectifier and the manufacturer must so make, grade and test the individual components of these rectifiers that they can be installed and forgotten. The paper suggests that we are not necessarily heading this way at the moment, and I should like the authors' comments on this point of view. How can we establish the parameters with which we are mutually concerned? We should not spoil an excellent concept by aiming for the minimum number of elements: it is much better to buy cells than auxiliaries.

Dr. W. G. Thompson: In common with many other forms of equipment, semiconductor rectifiers need ancillary apparatus to enable them to be worked closer to their limits, but much of the equipment used with these rectifiers is static. Some components, such as fuses, are expendable, while others, such as circuit-breakers, high-speed short-circuiting devices, fans and pumps, are active devices and it is necessary to ensure that the rectifier will still be safe in the event of failure or falling-off in performance of these active devices. It must be recognized that under internal fault conditions some semiconductor rectifier elements may be damaged. This is a startling concept but one which must be accepted in return for the improvements and economy which the semiconductor rectifier confers.

The paper refers to establishing voltage and loading ratings giving an acceptable life for the device. Can the authors give a clear indication of what they mean by the term 'acceptable life'?

In Section 3.3 the authors state that rated load current must not cause any significant deterioration of the rectifier within the life of the equipment, but I should welcome clarification of what the authors mean by deterioration.

Is the statement (in Section 4.2) that 'current sharing between strings is good, since the characteristics average' correct statistically in view of the small numbers of components?

The alternative to load-sharing devices for voltage or current when operating cells in series or parallel is to grade and match the cells, although grading at close intervals adds to problems of stocks and replacements. Derating factors for parallel operation must have economic limitations. A short-circuiter would permit fewer cells and reduced transformer reactance; is this Continental practice likely to be adopted?

Dr. J. C. Read: The paper is generally a summary of requirements and practice that are well known and widely accepted. I think, however, that high reactance in the transformer is more frequently useful than is suggested; it is one of the tools available to the designer for the purpose of ensuring that the fuses will not blow in case of d.c. short-circuit, and it is a particularly useful tool for this purpose, since it cannot detract from reliability. The objects of the high-speed short-circuiter can alternatively be achieved by merely providing an increased number of cells in parallel and/or higher reactance, and these methods appear to offer better reliability. Under what circumstances would the authors advocate omitting the rectifier d.c. circuit-breaker in large electrolytic installations above, say, 400 volts, where very large currents could be fed back from the d.c. busbars in case of fault?

(*Communicated*): It has been suggested that the term 'backfire' is an alien term from mercury-arc practice and should be replaced by some other word for semiconductors. Such a view fails to recognize the very close family relationship which exists between mercury-arc and semiconductor rectifiers. In both, the current flows because the charge carriers are immersed in an electrically-neutral ionized medium, for the ionized crystal lattice in the semiconductor is virtually a frozen plasma; in both, rectification takes place through the charge carriers in one direction being unable to surmount a potential barrier; the main causes of failure in both are the same, namely excessive contamination by impurities or excessive temperature; the mechanism of failure breakdown is practically the same in both, namely an avalanche process; and the consequences of failure are (with a difference of scale) the same in both, namely damage if the magnitude or duration of the resulting reverse current is too great. Several other parallels could be enumerated. When to this we add that both types of rectifier use the same circuits and for the same

applications, I think it is clear that every effort should be made to use a common terminology.

Mr. B. C. Hicks: I was interested to see the survival-current/time curves for a semiconductor rectifier shown in Figs. 3 and 4. Is there a maximum forward current which will saturate the current-carrying capacity of the semiconductor junction and beyond which the forward voltage will begin to rise above the ½ volt indicated in the text?

Table 1 refers only to medium-voltage fuses; are the authors aware that a high-voltage h.r.c. cartridge fuse which will limit the maximum transient arc voltage to 1·5 times the peak inverse voltage rating has been developed for semiconductor rectifier protection? The development of an h.v. fuse with this severe restriction on the maximum restriking voltage is an important step in the progress of protecting devices, since recent technical papers on this subject have indicated that a ratio of 1·7 was the best that could be achieved with h.v. fuses.

Table 1 refers to the performance characteristics of the special semiconductor rectifier fuses in terms of a prospective fault current with a power factor of 0·3 lagging. Is it essential in the design circuits to keep the power factor as high as 0·3 under fault conditions, or are the figures given capable of interpretation with fault currents having a power factor of less than 0·3?

The authors state that fuses are not used for the protection of rectifier cells against small overloads. In certain applications this principle is not followed by all rectifier engineers, and to meet their requirements fuses have been developed to conform to the general requirements of Section 7.1, but with the addition of a class-P fusing characteristic. Will the authors comment on this practice?

Mr. D. R. Coleman: It is apparent that the authors are mainly concerned with high-power germanium cells and their use in equipment rather than the smaller-power cells and even, perhaps, silicon cells. Nevertheless there is no mention of the protection necessary to deal with a rectifier open-circuit; we have observed this type of fault with some silicon rectifiers and it was not due merely to the passage of overload current. If open-circuit faults are borne in mind, the reader will question the validity of, and justification for, some of the authors' proposals. On the same question of applicability to all types of semiconductor rectifier, I suggest that the statement in Section 2.1 concerning breakdown voltage and the effects of junction temperature is not necessarily true for all types of device. The authors' intentions may be hidden, however, in their definition of 'breakdown voltage' from the previous Section.

The paper demonstrates the need for agreement on terminology. For example, in Section 2.3 the terms 'recurrent inverse voltage' and 'recurrent peak inverse voltage' are used; the sense suggests they have here the same meaning, but even that is not clear, since in the same paragraph there is a reference to surge voltages. Moreover, I wish to disapprove of the use of the term 'backfire' for semiconductor rectifiers. I accept it as a satisfactorily graphic term for mercury-arc rectifiers, because it indicates the cause of an inverse fault. Recognized methods exist for dealing with this cause to prevent a permanent damage effect, but it is equally clear that such clearance is not possible with a semiconductor rectifier. The authors point out that cell failure due to reverse degradation during the conduction period can lead to backfire currents after commutation. I should therefore prefer the term 'backfire current fault', since this is the effect and the cause cannot be removed.

In Section 2.2, describing the method of estimating junction temperature, there seems to be a fine distinction drawn between saturation current and leakage current. Have the authors employed the temperature-dependence of the forward characteristics for such an estimation to avoid those distinctions?

Section 4.2 refers to current sharing between cells. Would the authors agree that their statement 'current sharing between the strings shown in Fig. 5(*a*) is good' should preferably read 'current sharing with the circuit in Fig. 5(*a*) is better than with that in Fig. 5(*b*)'? We as manufacturers would recommend the use of graded cells for both those connections, the grading being primarily decided on the basis of dissipation inequality between cells.

In Section 4.3 the connection shown by Fig. 5(*b*) is said to 'allow detection of a single cell failure'. Should this not be 'allow detection of a parallel group failure' and is the detection of an open-circuit cell failure possible?

Is the variation of fuse characteristics with temperature of significance in the protection of semiconductor rectifiers?

Mr. W. J. Saysell: If with the aid of the paper and the published data one attempts to design a rectifier to supply, say, 12 kV at 2–3 amp, one does not obtain the cost advantage claimed in the paper; the authors are thus presumably dealing with high-current low-voltage work. The losses in these large series strings are much higher than with the conventional rectifier valve, and I should like the authors' views on this.

We shall be very fortunate if we can assume that the user will willingly derate his peak inverse voltage by 50%. If the user is told that his peak inverse rating is only a full rating and the recommended rating is only one half of this, how can one be sure that he will keep to what he is told is good design practice?

Can any indication be given of the additional cost of grading?

Mr. J. M. Waddell: An important aspect of the techniques described in the paper is satisfactory communication between cell manufacturer and user: the user may buy his cells (especially the smaller ones) from several manufacturers, and a committee with which I am associated is endeavouring to establish a common basis for the expression of characteristics and ratings. For example, Figs. 3 and 4 are both forward-current survival curves, but the ordinate in Fig. 3 is mean current while that in Fig. 4 is peak current, while fuse characteristics are usually plotted in terms of r.m.s. current. I should like the authors' views on how survival curves should be given, especially for reverse surges, where allowance must be made for the voltage, power, moment of application and duration of the surge.

In Section 2.3 the authors say that 'the ratio of recurrent peak inverse voltage to breakdown voltage can be chosen to suit different applications', but they do not say what sort of ratios are usual; my experience is that ratios from 1·2 to 4 have been used successfully. Does the ratio appropriate to silicon differ from that for germanium?

For many silicon rectifiers (such as that shown in Fig. 1) a rise of junction temperature leads to an increase of breakdown voltage, and not a decrease as stated in Section 2.1. The junction temperature of silicon rectifiers is readily measured by techniques similar to those described in the paper for germanium except that, for the reasons given in Section 2.2, the reverse leakage current cannot be used, and the forward leakage current at a bias of about 0·4–0·5 volt is measured instead. A similar theoretical relation then holds.

I, too, dislike the term 'backfire fault' in this context and suggest 'cell short-circuit'.

In reply to Mr. Hicks, there is normally a cut-off current to curves of the type shown in Figs. 3 and 4. This may be due to the sudden increase of forward voltage drop which can occur at very high current densities as a result of hole-electron scattering. Alternatively, considering the thermal equivalent circuit of the rectifier as a series string of low-pass filter sections, very short transients are restricted to the initial section, whose low thermal capacity causes a large temperature rise for a small energy input.

Mr. R. A. A. Newman: My experience is that the reverse

breakdown voltage of silicon diodes rises with temperature in many instances and is largely dependent upon the materials and processing used in manufacture.

Reverse-voltage waveform spikes caused by the flow of carrier storage current and the input circuit reactance never appear in practice as large as those illustrated in Reference 12, and the effect is much less noticeable with silicon than with germanium. Manufacturing techniques have improved considerably since 1955, and in many cases the attitude to protection against these spikes would appear to be over-emphasized.

Voltage surges occurring on domestic mains supply often embarrass the manufacturer of small semiconductor devices, since they may adversely affect, for example, the performance of silicon diodes in a television receiver. Very little information is available from supply undertakings on such surges, which can arise from a multiplicity of causes and vary from area to area. Surges shorter than 0·1 millisec are relatively easy to suppress economically, but those lasting 2 or 3 cycles are extremely difficult. Have the authors any views on this problem?

A circuit used in America and elsewhere for surge protection comprises a bridge rectifier feeding a capacitor fitted with a discharge resistor. The capacitor charges to the peak of the normal voltage waveform, and surges are absorbed by charging it to a slightly higher voltage, thus suppressing the surge voltage appearing on the protected equipment. The circuit does not seem to be very widely used, but appears to have many attractive advantages. Have the authors any experience in its use?

Mr. F. F. Roberts: I should have liked to see in the paper some discussion of the determination, by measurements on the complete cell, of the essential parameters of the thermal analogue circuit which is briefly mentioned in connection with Fig. 2, and which is considered synthetically in Reference 9. The equipment designer needs to know more about the thermal equivalent circuit and to have some idea of the thermal time-constants involved. Fig. 2 implies that there is at least one important thermal time-constant of the order of milliseconds, but there is also another of the order of minutes, or even up to an hour or so, associated with the way in which the cell is connected to its heat sink. One of the reasons for the difference between the overload capabilities of semiconductor cells and those of the older (selenium and copper oxide) types is that the sum of the thermal resistances associated with the short time-constants contributes, in the new cell types, a larger proportion of the total thermal resistance between the junction and the cooling medium.

In Fig. 3 I would question the significance of the description of curve (*a*) as a constant-coulomb line. This description is permissible only if the voltage drop is independent of current, whereas, in fact, at high enough currents the forward voltage drop will increase. The equipment designer is greatly interested in knowing what will happen to the forward voltage drop under surge-current conditions.

Will the authors comment on voltage sharing between cells in series when their hole-storage characteristics differ? In Section 6.1, can more details be given of the thermal equivalent circuit, or the essential thermal resistances and thermal time-constants, for the fuses? Section 7 begins with the words 'The foregoing discussion has shown'. I wonder whether something has been omitted in the preparation of the paper, for I should like to see some discussion. What is the significance of the last column of Table 1?

Mr. K. L. Streeter: I think that Mr. Broughall's fears about the complications of circuit protection, particularly in connection with traction, will eventually be removed. Semiconductor suppliers have investigated the performance of their device in considerable detail, primarily because of its thermal mass and its sensitivity to voltage surges. We should now like to know the sort of surges produced in traction overhead supply lines and the demands of the circuit which forms the load on the rectifier. It is difficult to answer the surge question, and the starting characteristics of traction motors and the duties imposed on them are not easily assessed, but I believe these will in time be known and that protection for traction rectifiers will not include all the devices currently required.

Short-circuiting switches have hitherto been used primarily for the contact rectifier, a typical operating time being 2 millisec. But with reasonable rectifier-transformer reactance the switch need not operate so rapidly for semiconductors and a different design of switch could be investigated.

While I agree that monocrystalline rectifiers have a very small overload capacity, I wonder who drew the first overload curve and suspect that it came from America. The first impression has caused much discussion, for an overload is relative only to the chosen normal load. Who established the normal load? Because there was some optimism in the first curves presented in terms of normal current and working reverse voltage, we are in a season of pessimism of the performance of the monocrystalline rectifier. But with greater application knowledge and sensible ratings, coupled with improvements in the semiconductors themselves, much of this complex overload protection can disappear and the rectifier itself will show the advantages which had been hoped for.

Mr. H. W. Baxter (*communicated*): Will the authors amplify Section 7.2 by giving details of the construction of the fuses. These appear to operate in about 10 millisec with a 100% overcurrent and I should like to know whether this is achieved by running a silver element at almost its melting temperature or by other means.

Mr. K. Dannenberg (*communicated*): The authors refer in Section 6 to the prediction of protection requirements with regard to the energy problems in relation to fuse operation in conjunction with the protection of semiconductor rectifiers.

The authors have rightly stressed the importance of restraining the voltage spikes which may occur on fuse operation. This is a specific problem with high-voltage fuses but has been successfully solved. High-voltage-fuse technique in Britain has been well in advance of the work and developments carried out on the European and American continents by restricting the arc voltage to 1·5 times the peak recovery voltage.

No mention is made in the paper of the possible influence of capacitance in the circuit; its influence on the natural frequency of the circuit may have some considerable importance in regard to fuse performance, especially at higher voltages. Comprehensive investigations have recently been completed showing how this problem can be solved.

These protective fuses, in common with all conventional h.r.c. fuses, are non-deteriorating and unaffected by ambient temperature within the specified limits of each individual design; moreover, fuse-link ratings down to 6 amp are available for all voltages.

It should be stressed that surge protective devices as they are applied to semiconductor rectifiers are not only physically small but do not require servicing. They are self-protecting in so far as they incorporate either disconnection devices or pressure-relief diaphragms which safeguard them should, for unforeseen circumstances, they be overstressed relative to their maximum surge-energy duty.

[The authors' reply to the above discussion will be found overleaf.]

THE AUTHORS' REPLY TO THE ABOVE DISCUSSION

Messrs. D. B. Corbyn and **N. L. Potter** (*in reply*): The paper mainly relates to large rectifier equipments, but the principles apply universally.

Like Mr. Broughall, we desire simplicity. Protective devices are static components, simpler than mercury-arc auxiliaries, and their complete elimination entails the use of more rectifier cells. The parameters of special importance in semiconductor traction equipment are peak load, duty cycle and highest likely impulse voltage on the catenary.

In answer to Dr. Thompson, continuous operation of semiconductor rectifiers at too high a junction temperature causes reverse deterioration. The maker of industrial cells must carry out life tests and define a rating not causing noticeable deterioration in, say, 25 years.

No change of forward voltage drop has been noted with germanium or silicon rectifiers. An occasional string showing reverse breakdown is disconnected by string fuses and can be replaced. With two or more series cells per string the worst possible current distribution between strings is unchanged, but its probability decreases.

The choice between derating cells for parallel operation and using load-sharing devices is mainly economic.

With Dr. Read we regard the term 'backfire' as an excellent descriptive term.

Reactance is a useful tool but carries the penalty of worse regulation and power factor, possibly needing a regulator and less efficient transformer. Where the risk of terminal short-circuits is very small low reactance seems justified. Short-circuiter protection is cheaper than additional cells for large equipments but more expensive on small equipments. The choice is made on economic grounds.

We consider the omission of rectifier d.c. circuit-breakers permissible if h.r.c. fuses prevent cell failures short-circuiting the d.c. busbars and if the busbars are constructed to eliminate the risk of short-circuits. In large installations a busbar fault on one equipment should not cause loss of fuses on nearby equipments, although these will necessarily trip. In such circumstances the d.c. circuit-breaker is an expensive insurance against the remote chance of a short-circuit on busbars between cells and isolators. The question becomes one of economic risk, and the nature of the whole process must be considered.

In answer to Messrs. Hicks, Waddell and Roberts, Fig. 3 shows the theoretical survival curve of a perfect rectifier with constant forward voltage drop. In practice forward voltage drop increases at high over-currents and actual survival curves allow for this. Only curves of cell and fuse survival in peak current have any meaning below one cycle. We have not yet used the high-voltage semiconductor fuse to which Mr. Hicks refers, but are very interested. The lagging power factor of $0 \cdot 3$ is typical, not obligatory, but if the pre-arcing I^2t of the fuse is available, cut-off currents can be calculated for other conditions. Cell fuses are acceptable overload protection on small equipments only; a.c. and d.c. fuses are usable but discrimination with cell fuses is difficult.

The risk of open-circuited cells, mentioned by Mr. Coleman, is remote in well-made cells. Reverse leakage current and forward voltage drop have been used to estimate junction temperature. The leakage current at low reverse voltage is a kind of saturation current and is not proportional to voltage.

The variation of fuse characteristics with ambient-temperature changes is negligible. Mr. Saysells points out that a silicon rectifier for 12 kV is less efficient than a mercury-arc device: improvement of silicon cell characteristics will eventually change this, and the silicon controlled rectifier will provide grid control. Lower voltage safety factors are usable provided that over-voltage protection is possible with certainty.

Theoretically the reverse-voltage/survival-time curve suggested by Mr. Waddell is obtainable, but we would hesitate to draw this at the moment.

Voltage safety factors a little above two for germanium and silicon are common, being limited by fuse arc voltage and lightning arresters.

Answering Mr. Newman, we find that the reverse voltage breakdown of silicon cells always decreases at very high temperatures, and also that the hole storage is not very serious with some silicon cells.

We should not expect mains-voltage surges of 2 or 3 cycles to exceed 30% over-voltage. The circuit mentioned by Mr. Newman appears similar to one mentioned in Reference 15 and tests showed that it offered no great advantages.

Mr. Roberts will find further details of the thermal analogue in Reference 9. The junction time-constant is of the order of milliseconds for fault conditions. For overloads the whole thermal system including the cooling fins is important and usually has a time-constant of a few minutes for forced-air cooling. For oil-immersed cells the total thermal time-constants for overloads may be 30–60 min.

In short strings poor voltage sharing due to hole-storage effects has been found unimportant; for long strings capacitive voltage-dividers appear necessary.

The fuse designer needs to study detailed thermal characteristics, but survival curves and the details given in Table 1 are satisfactory for designing rectifier protection. The preamble to Section 7 refers mainly to Section 6. The last column of Table 1 is added because a rectifier cell may fail for excessive peak-let-through current, $\int i^2dt$ or coulombs. We examine all quantities in equipment design.

In answer to Mr. Streeter, lightning gives the worst voltage surges on traction systems; one should assume that the surge voltage reaches the line impulse level and design accordingly.

For a traction rectifier it is satisfactory to obtain the worst possible acceleration currents and 'notching' curves for the motors. Slowing down the short-circuiter operation has little point, since this eliminates its main advantage.

The new semiconductors run at high current densities and thus are never likely to show the high overload capacity of selenium rectifiers, which necessarily run at much lower current densities. With germanium or silicon cells the use of a current density comparable with that of a selenium plate would greatly increase the size of the semiconductor equipment. For these reasons we never expect the overload capacity of the new semiconductors to equal that of the selenium rectifier.

Voltage-surge-protection circuits are cheap, small and reliable; they are likely to remain for high-voltage equipments, but will be simplified for low voltages since the peak inverse voltages of the cells will improve.

Answering Mr. Baxter, the fuse is specially constructed with a carefully shaped silver element not run near its melting-point and not subject to ageing.

We thank Mr. Dannenberg for his interesting comments on fuse performance.

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THE APPLICATION OF POWER TRANSISTORS TO THE OPERATION OF GAS-DISCHARGE LAMPS FROM D.C. SUPPLIES

By I. F. DAVIES, B.Sc., Associate Member, and D. DUNTHORNE, B.Sc.(Eng.), Graduate.

(*The paper was first received* 26*th March, and in revised form* 30*th October*, 1959. *It was published in January*, 1960, *and was read before the* UTILIZATION SECTION 14*th January*, 1960.)

SUMMARY

The operation of fluorescent tubes on high-frequency (up to 20 kc/s) supplies is considered. The characteristics of sine-wave and square-wave transistor inverters are discussed, and a preference is expressed for sine-wave supplies for lamp operation. Design details and component characteristics for sine-wave lamp inverters are considered and various types of circuit are mentioned.

(1) INTRODUCTION

Subject to certain limitations, power transistors may be used in conjunction with most types of gas-discharge lamp, but the present paper is confined to a discussion of their use in relation to the low-pressure hot-cathode fluorescent tube.[1–3] Up to recent times the operation of fluorescent tubes from d.c. supplies was attended by a high ballast power dissipation and loss of light output due to electrophoresis—the migration of positive mercury ions to the cathode[4, 5]; furthermore, it was impossible to operate these lamps directly from d.c. supplies as low as 24 volts, and for such applications it has been necessary to use d.c./a.c. inversion equipment. Rotary inverters,[6] vibrators[7–11] and, recently, mercury-jet inverters[12] have all been used, mainly with fluorescent tubes in public-transport vehicles, but all suffer from the inherent wear and maintenance problems associated with moving components. Such systems, however, operate the lamp on an alternating supply and hence avoid the disadvantages of d.c. operation.

During the last two years the development of self-driven inverters, using germanium power transistors, has eliminated all moving components. Transistor inverters can be designed to operate from any d.c. supply from about twelve up to several hundred volts. These inverters can be associated with a single fluorescent tube and ballast or can be constructed as a power pack to supply a.c. power to a number of separate lamp-and-ballast combinations.

The paper considers the design of such inverters and gives details of recent installations.

(2) THE OPERATION OF FLUORESCENT TUBES ON HIGH-FREQUENCY SUPPLIES

(2.1) Sine-Wave Voltage Supply

Circuits for the operation of fluorescent tubes at frequencies up to 1 kc/s have been in use for many years, mainly in transport installations.[13] As the operating frequency is increased, some of the lamp parameters change. This is especially true of the lamp voltage, which changes from a square to a sine waveform.[14] This is brought about by the ionization in the lamp approaching a steady value as the frequency increases, and thus causing the lamp dynamic characteristic[15, 16] to approach a straight line at frequencies of 5 kc/s and greater. With a sine-wave lamp current, as is obtained with a choke ballast below 1 kc/s and with choke and capacitor ballasts above this frequency, this results in an increased lamp waveform power factor and an increased luminous efficiency.[5, 16] Other contributing factors are lower cathode losses and greater efficiency of production of the ultraviolet radiation which excites the lamp phosphor. There appears to be little advantage in using supply frequencies greater than about 20 kc/s, since the efficiency/frequency curve levels off at this point.[16] Few data on lamp life performance are available, but it is not expected that lamp life will be worse at high frequencies than at 50 c/s. Meyers and Strojny[14] have claimed recently that the radio-frequency interference generated in fluorescent tubes[17] is much reduced at higher operating frequencies, especially where continuous cathode heating is employed.

An adverse effect of the use of high-frequency supplies is the increased voltage required to start the lamp. This has been recorded by Meyers and Strojny,[14] and the authors can confirm the results with the 4 ft 40-watt hot-cathode fluorescent tube. Lamp starting is, of course, affected by the nature of the tube surface, humidity, temperature and the presence or otherwise of external conductors. These parameters have been discussed elsewhere for 50 c/s operation,[18–23] and it is probable that the magnitude of their effect is similar at higher frequencies. The physical size of the lamp ballast decreases as the supply frequency increases. This is advantageous, especially in transport lighting installations where space is at a premium. In general, the ballast cost will be reduced, although at the highest frequencies special ballast design features may offset this.

(2.2) Square-Wave Voltage Supply

The authors have little experience of the operation of fluorescent lamps having a square-wave lamp current, from square-wave voltage supplies. However, Campbell, Schultz and Kershaw[16] report that the lamp efficiency at a given frequency under these conditions is higher than with sine waves. No life data are available for lamps operating with square current waves.

(3) SELF-DRIVEN SQUARE-WAVE AND SINE-WAVE TRANSISTOR INVERTERS

(3.1) Square-Wave Inverters

A typical push-pull self-driven square-wave inverter is shown in Fig. 1; this unit uses the transistors in a switching mode[24] and the inverter transformer is designed so that its magnetic core saturates during parts of the operating cycle. Design considerations have been discussed elsewhere[25–27] and will not be considered here.

Operation with a purely resistive load results in the collector voltage and current waveforms shown in Fig. 2(*a*), and the switching diagram, which is a plot of these two parameters, is shown in Fig. 3(*a*). These waveforms are well known, but it should be noted that the collector voltage never exceeds twice the steady direct supply voltage. When the load is made

The paper is an official communication from the Staff of the Research Laboratories of The General Electric Company Ltd., Wembley, England.
Mr. Davies is now with Philips Electrical Ltd.

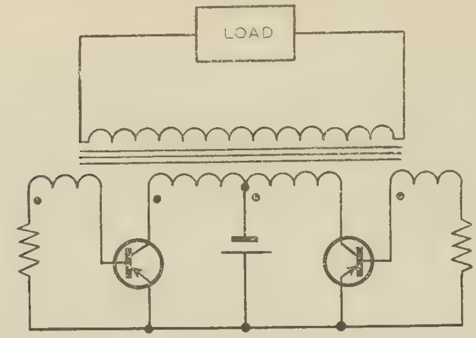

Fig. 1.—Typical push-pull square-wave inverter.

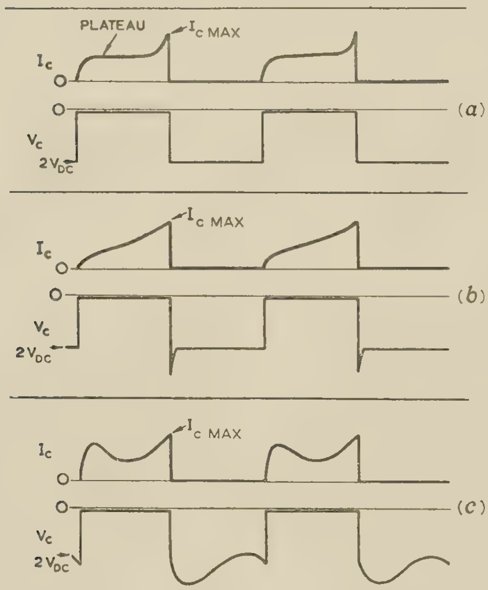

Fig. 2.—Transistor waveforms for a square-wave inverter.

(*a*) Resistive load.
(*b*) Inductive load.
(*c*) Capacitive load.

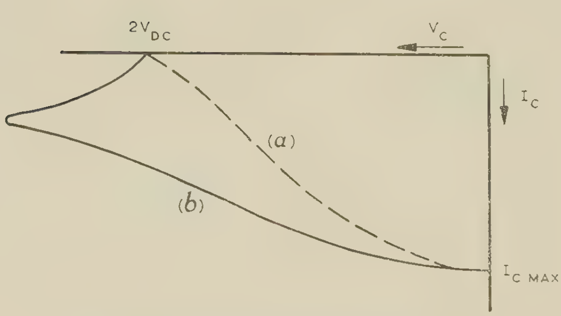

Fig. 3.—Square-wave inverter switching diagrams.

(*a*) Resistive load.
(*b*) Inductive load.

reactive, however, the transistor operation is changed. With an inductive load the waveforms in Figs. 2(*b*) and 3(*b*) are obtained, and it is seen that the peak collector voltage is well in excess of twice the supply voltage. This, of course, places a lower limit on the transistor voltage specification. When the load is made capacitive the collector current and voltage waveforms shown in Fig. 2(*c*) are obtained. It will be seen that the leading edge c the collector-current waveform is now raised above the centr plateau, and additional care must be taken to ensure that th height of the trailing edge, which is governed by the driv current and transistor current gain, remains above the leadin edge. If this does not happen the transistors will cease to switc and the inverter will oscillate approximately sinusoidally at much higher frequency. The switching diagram for this inverte is not shown, since it may take many forms depending on th ratio of capacitive to resistive load and the inevitable leakag inductance which will be present. Likewise, the collector voltag waveform can take many forms depending on the amplitude an frequency of the ringing oscillation which will occur whe square-wave voltages are applied to circuits containing bot inductance and capacitance.

If the load resistor is replaced by a discharge lamp, havin amongst other things a negative resistance characteristic, th circuit operation is even more complex, and so far efforts t operate this type of load with either an inductive or capacitiv ballast, from a square-wave inverter, have been only partiall successful.

(3.2) Sine-Wave Inverters

The circuit of a typical class-B sine-wave integral inverter i shown in Fig. 4, and the associated transistor waveforms ar

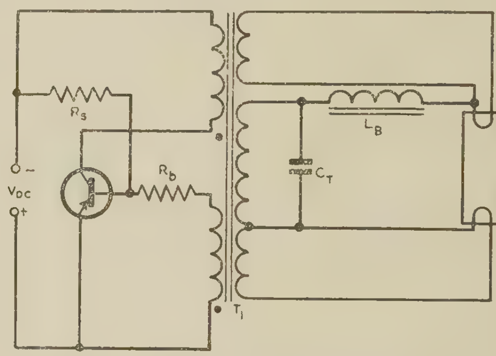

Fig. 4.—Single-ended sine-wave inverter.

shown in Fig. 5. The inverter is a self-driven tuned-collecto oscillator of a type which has been adequately describe elsewhere,[28, 29] and the load consists of a switchless-sta fluorescent-tube circuit having uncompensated lamp cathod heating derived from windings on the oscillator transforme The overall theoretical efficiency of such a circuit is 78%, but i practice the efficiency is between 50 and 55%. The usual metho of defining such an efficiency is

$$\frac{\text{Lamp arc power} + \text{Lamp cathode power}}{\text{Total d.c. input to circuit}} \times 100\,\%$$

A better efficiency is obtained by driving the oscillator int class-C conditions. This may be most conveniently done b connecting a suitable capacitor across the base resistor, R_b.

The degree of class-C operation may be increased, with corre sponding increases of efficiency, until the peak collector curren which will increase with decreasing conduction angle, reache the maximum specified for the transistor.

A simplified equivalent circuit of the secondary side of trans former T_1 is shown in Fig. 6. The lamp is replaced by a resisto R_L, and the cathode heating load is represented by a transferre resistor, R_F. The tuning capacitor, C_T, is split into two parall capacitors, of which C represents the capacitance necessary t correct the circuit comprising the lamp and its ballast L_B t

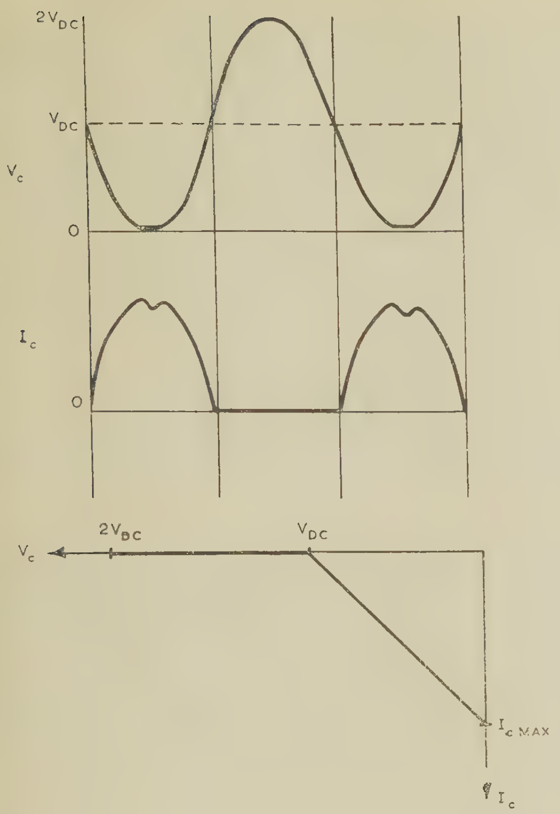

Fig. 5.—Transistor waveforms for class-B sine-wave inverter.

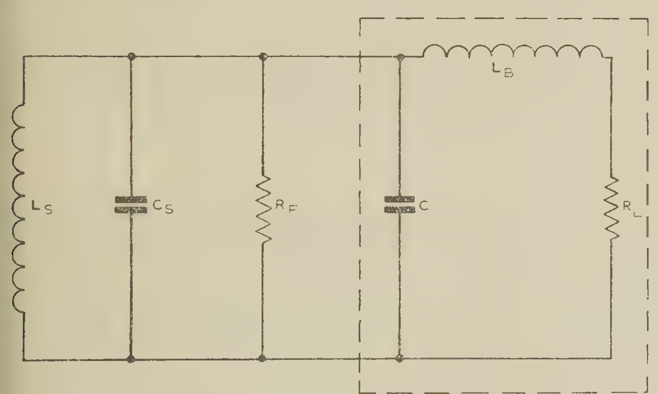

Fig. 6.—Equivalent output circuit for a sine-wave inverter.

unity power factor. The dynamic resistance of this part of the circuit, L_B/CR_L, may be added to the cathode load, R_F, to give R_T, the total load resistance of the circuit, where

$$R_T = \frac{R_F L_B}{L_B + CR_L R_F}$$

It can now be shown that, for the secondary circuit,

$$Q = \omega C_s R_T.$$

The values of Q and operating frequency are chosen from design considerations which will be discussed later, but in general, for any particular frequency there is an optimum value of Q for maximum efficiency.

(4) DESIGN CONSIDERATIONS AND COMPONENTS OF PRACTICAL SINE-WAVE INVERTERS

(4.1) Transistors

The power transistors which have so far been available in production quantities in this country at a price suitable for commercial applications have been *p–n–p* alloy junction germanium transistors, and unless otherwise stated, the paper deals with inverters employing this type of transistor.

(4.1.1) Transistor Characteristics and Measurements

The transistor characteristics required are briefly as follows:

(*a*) High peak and mean current ratings.
(*b*) Low internal base resistance.
(*c*) Low intrinsic thermal resistance.
(*d*) A high maximum junction temperature (in this respect silicon transistors are preferable to germanium).
(*e*) A high cut-off frequency, f_{hfb}.
(*f*) High reverse-voltage characteristics.

The last is essentially a combination of three parameters, namely V_s, $V_{cb\,max}$ and 'punch through'. Since these parameters and their measurement have been dealt with extensively elsewhere,[30] only a brief mention will be made here.

V_s is the avalanche breakdown voltage occurring between the emitter and the collector with the base open-circuited. As the collector-emitter voltage, V, is increased, the common-base gain, h_{fb}, increases according to

$$h_{fb} = h_{fb0} \times \frac{1}{1 - \left(\frac{V}{V_b}\right)^3}$$

where V_b is the collector-base avalanche breakdown voltage and h_{fb0} is the common-base current gain at low collector voltage.

It is thus seen that when $V \rightarrow V_b$, $h_{fb} \rightarrow \infty$. However, before this happens a value of V is reached at which $h_{fb} = 1$, and hence the common-emitter gain, h_{fe}, approaches infinity, since

$$h_{fe} = \frac{h_{fb}}{1 - h_{fb}}$$

The emitter-collector voltage at which this occurs is V_s.

$V_{cb\,max}$ is the reverse characteristic of the base-collector junction and is fixed arbitrarily as the voltage at which the collector leakage current, I_{c0}, at a specified ambient temperature reaches a specified upper limit, e.g. 1 mA at 25° C.

If at the same time as a reverse voltage is increased across the base-collector junction, the voltage across the base emitter junction is observed, a point is reached when a sudden increase of this voltage occurs. This happens when the base-collector space-charge region extends to the emitter junction and is called 'punch-through'.

For a self-driven sine-wave oscillator V_s should be greater than the battery voltage, since at the part of the cycle at which the base is effectively an open-circuit, i.e. at the point where the feedback voltage passes through zero, the collector-emitter voltage equals the battery voltage.

When the transistor is non-conducting the collector-emitter voltage will normally rise to twice the battery voltage, and in certain cases, e.g. single-ended inverters, to greater than twice the battery voltage, and thus both $V_{cb\,max}$ and punch-through must exceed this value.

(4.2) Lamp Ballasts

(4.2.1) Inductive Ballast.

It will be seen from Fig. 6 that when the lamp is alight the frequency of oscillation of the inverter is determined by resonance between L_s and C_s. However, before the arc has

struck the frequency is determined by L_s and $C_s + C$, and hence the frequency of operation is lower during lamp starting.

Any practical inverter will have to cater for a range of input voltages; for example, a nominal 24-volt lead-acid battery, such as is encountered in aircraft lighting, will have a voltage range of 22–30 volts, with a working nominal of 28 volts. As the input voltage is reduced from this value the frequency of the inverter will fall slightly, and hence lamp power variations will be smaller than would be the case with constant frequency. This, of course, means that, to ensure satisfactory inverter operation at all direct supply voltages, the base resistance must be adjusted at the lowest direct voltage; however, some overdrive will occur as the direct voltage is increased.

An inductive ballast has two main advantages over a capacitive ballast:

(*a*) When an inverter is being operated at a frequency at or above the transistor cut-off frequency, it is preferable for the frequency to fall in the lamp-starting condition rather than to rise, as it does with a capacitive ballast.

(*b*) The lamp current waveform is sinusoidal; this is especially important with 'single-ended' inverters.

(4.2.2) **Capacitive Ballast.**

A capacitive ballast used in this type of circuit has two drawbacks, which are corollaries to the points discussed in Section 4.2.1; these are:

(*a*) The frequency of the oscillator rises when the lamp is in the starting condition, and this may lead to gain troubles if the transistors are operated at or near their cut-off frequency.

(*b*) Any harmonics in the inverter output voltage will become magnified in the lamp-current waveform.

Against the above disadvantages a capacitive ballast will generally be lighter and cheaper than the equivalent inductive ballast and may be preferable in some installations.

(4.2.3) **Resistive Ballast.**

Although cheap and light, a resistive ballast is generally not considered because of the large power loss involved. The voltage needed to start a fluorescent lamp under the worst conditions of supply voltage and temperature is about 2·5 times the running voltage of the lamp. This means that the power loss in a resistive ballast will be at least twice the power in the lamp.

(4.3) Transformers

Apart from the usual design considerations of flux density of the core and current ratings of the wire used, the most important factor in the design of an inverter transformer is the leakage inductance between the primary winding, the feedback windings and the secondary winding across which the tuning capacitor is connected. If appreciable leakage inductance exists, it is possible for the inverter to oscillate simultaneously at two different frequencies; the frequency of the main oscillation is determined by L_s and C_s, and a higher frequency is superimposed on this wave, determined by C_s and the leakage inductance. The design considerations for a transformer of minimum leakage inductance have been given elsewhere.[31]

In order to obtain the two requirements of frequency of operation and Q-factor of the secondary tuned circuit, it is generally necessary to have an air-gap in the transformer core. This will normally preclude the use of toroidally wound transformers for sine-wave inverters.

(4.4) Capacitors

When choosing capacitors for these applications, full allowance must be made for the limitation imposed by supply frequency. Catalogues of paper-dielectric capacitors commonly list them under direct-voltage ratings and allow up to 10% superimposed alternating voltage at a maximum frequency of 100 c/s. From these figures it is impossible to deduce the permissible working voltage at, say, 5 kc/s. The loss factor of such capacitors at the stated 100 c/s may be about 0·005, but it increases rapidly with a power of the frequency which is generally about 1·5–2·0. The power loss in a capacitor is given by $\omega CV^2 \times$ loss factor, and for a 0·5 μF capacitor rated at 500 volts d.c. plus 50 volts of 100 c/s, and operated on a 50-volt 100 c/s supply with or without a d.c. component, the loss is about 0·004 watt. At 50 volts and 1 kc/s this rises to 0·15 watt, and the capacitor would still keep quite cool; however, at 10 kc/s the loss rises to over 4 watts, and overheating of the capacitor might occur, together with a marked reduction in life.[32]

(4.5) Frequency of Operation

The choice of operating frequency will depend on such considerations as lamp luminous efficiency (see Section 2.1), size of components and cost, characteristics of available transistors and noise.

(4.5.1) **Size of Components and Cost.**

The higher the frequency the smaller generally will be the wound components. This is important in aircraft lighting, where weight is a major consideration. For frequencies up to about 1 kc/s, silicon-iron cores may be used in chokes and transformers, and these are relatively cheap. Above 1 kc/s, more expensive nickel-iron cores are needed, in order to reduce the iron loss. At about 2 kc/s, ferrite cores become a practical proposition.

(4.5.2) **Transistors.**

With the power transistors at present available the maximum frequency of operation is 6–8 kc/s in this type of inverter with the transistors working at their maximum peak collector-current rating. For a lower-power inverter it may be possible to use a transistor with a higher peak collector-current rating than is required, thus obtaining a higher gain at the peak currents used and a usable gain at frequencies in excess of the cut-off frequency. Using this principle, inverters have been designed to operate at frequencies up to 25 kc/s.

(4.5.3) **Noise.**

Wound components in an a.c. circuit will give rise to a certain amount of noise,[33] and this may be objectionable. For well-designed and tightly clamped components operating at frequencies up to a few hundred cycles per second, noise is not difficult to suppress, but some special form of noise suppression is needed with nickel-iron cores, owing to the marked magnetostrictive effect of this class of material. Potting materials such as castor-oil-urethane[34] are suitable for this purpose. Provided that it is within the capabilities of the transistors used, the frequency of operation may be taken above the limit of human audibility, i.e. 16–18 kc/s. The effect on animals, however, may still be apparent.

(4.6) Thermal Considerations

The rapid dissipation of heat generated in the transistor is a problem ever present with the designer of equipment using power transistors. The dissipation in the transistor is greatest at the base-collector diode, and for this reason the collector in power transistors is mounted directly on to a large mass of metal, usually copper.[35, 36] Heat from the collector junction passes by conduction into the copper and thence into some form of heat dissipator, which also serves as a suitable mounting for the transistor. A maximum junction temperature, T_{jmax}, at which the transistor may be operated safely is specified by the

manufacturer, and the designer must ensure that this is not exceeded at the highest ambient temperature in which the equipment is to be used.

In considering the design of a practical heat dissipator, or heat 'sink' as it is frequently called, it is convenient to compare heat flow and electric current flow and also to use the concept of thermal resistance,[37–40] which is analogous to ohmic resistance. The thermal resistance of a transistor and heat-sink combination is defined as

$$\theta = \frac{T_j - T_A}{P} \text{ degrees C per watt}$$

where T_j is the collector junction temperature at an ambient temperature of T_A when a power P is dissipated in the transistor. The maximum thermal resistance which can be tolerated in order to keep the junction temperature at or below $T_{j\,max}$ is

$$\theta = \frac{T_{j\,max} - T_{A\,max}}{P} \text{ degrees C per watt}$$

The thermal resistance, θ, can be considered to consist of several components (see Fig. 7). θ_i, the intrinsic thermal resistance between the transistor junctions and the mounting surface, is fixed by the transistor designer and is usually about 1·5–2 deg C/watt; θ_m represents contact resistance at the mounting surface, and is normally not greater than 0·3 deg C/watt; θ_r allows for loss of heat by radiation from the transistor container which is only a few per cent of the total dissipation; θ_k and θ_c represent heat-flow paths in and out of the heat sink by conduction and convection respectively. If the transistor must be insulated electrically from other circuit components, this can be done by the use of insulating materials, such as mica, between the transistor mounting base and the heat sink. This introduces a further thermal resistance at the points XX in Fig. 7. Alternatively, the transistor and heat

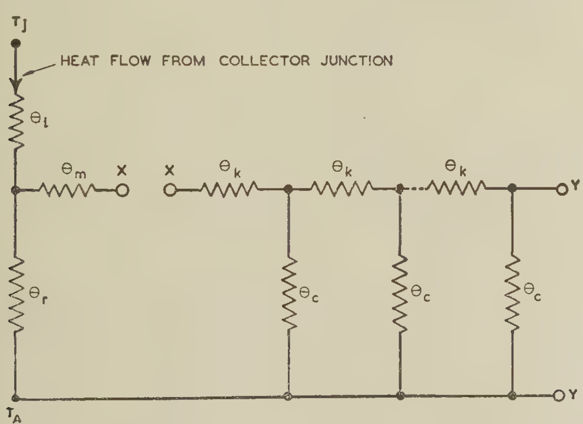

Fig. 7.—Thermal-resistance network of a transistor and heat sink.

sink can be in electrical and mechanical contact and the whole assembly isolated electrically, when the thermal resistance appears at YY. It is immediately evident that the latter method introduces less thermal resistance.

With the temperatures involved with the average power transistor, a heat sink which dissipates most of its absorbed heat by convection has been found to be the most satisfactory. The thermal resistance of such a heat sink is inversely proportional to its surface area, and for maximum efficiency all its surface should be at the same temperature.

Typical heat sinks are shown on the inverter in Fig. 15 indicating the method used to obtain a large surface area in a small volume. The type and surface of the metal used have only a

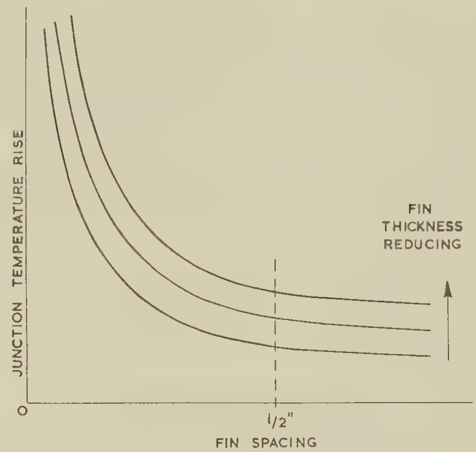

Fig. 8.—Variation of transistor junction-temperature rise with fin spacing.

small effect on the efficiency of the sink, but the fin thickness and spacing must be carefully considered. Fig. 8, which illustrates this, shows that a fin spacing, d, of less than about $\frac{1}{2}$ in is detrimental to heat sink efficiency, owing to the presence of Langmuir layers of air on the fin surfaces.[41–43]

Since θ_i is about 1·5–2·0 deg C/watt, little advantage is gained in making heat sinks having a thermal resistance of less than about 0·5 deg C/watt.

(5) PRACTICAL LAMP-INVERTER CIRCUITS

(5.1) Sine-Wave Circuits

(5.1.1) Single-Ended Inverter.

The basic single-ended common-emitter tuned-collector inverter has already been described in Section 3.2. Where the negative side of the supply is earthed, a configuration which permits the transistor to be bolted directly to an earthed metal plate without the use of insulation washers is used. This is the common-emitter earthed-collector circuit,[44] and a typical class-C inverter of this type is shown in Fig. 9. The inverter is

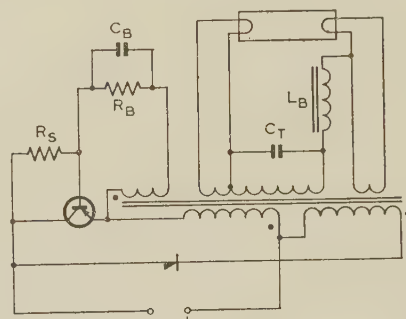

Fig. 9.—Common-emitter earthed-collector sine-wave inverter with spill-over circuit.

shown including a 'spill-over' circuit which consists of a winding, of an equal number of turns to the primary winding, connected in series with a rectifier across the d.c. supply.[45] If the lamp should become disconnected from the circuit, the energy supplied to the transformer core on the conducting half-cycle would give rise to an excessive voltage swing on the non-conducting half-cycle, damaging the transistor. The purpose of the spill-over circuit is to return this energy to the supply. If the spill-over

circuit is not used the no-load voltage swings in the inverter may be reduced by judicious choice of the secondary-inductance/tuning-capacitance ratio. A value of 2 for the secondary-circuit Q-factor has been found to give about optimum inverter efficiency. If this is increased, two beneficial results occur, at the expense of efficiency, namely

(*a*) The no-load voltage swings are reduced.
(*b*) The increased flywheel effect gives a more symmetrical lamp-current waveform when using the single-ended inverter.

(5.1.2) Push-Pull Inverter.

The circuit of a typical push-pull sinusoidal inverter[46,62] is shown in Fig. 10; it has a common biasing circuit, C_B, R_B,

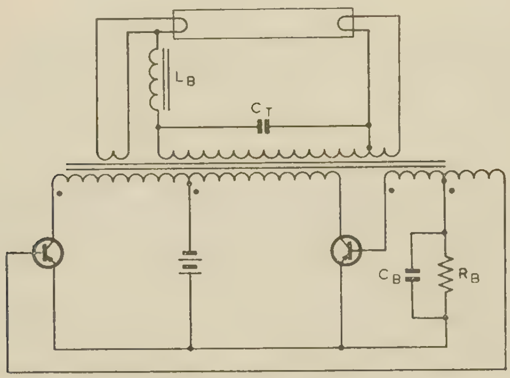

Fig. 10.—Class-C push-pull sine-wave inverter.

which means that the transistors must be fairly well matched in gain. A greater variation of gains can be tolerated at the expense of extra components by using two separate biasing circuits, one in each transistor base lead. The advantages of push-pull over single-ended operation are

(*a*) Twice the power output for a given direct supply voltage.
(*b*) Improved lamp-current waveform, owing to the greater symmetry of the circuit.
(*c*) The load may be disconnected without excessive voltage swings occurring.

(5.1.3) Series Operation of Transistors.

The power transistors available in this country are designed for sine-wave inverter operation on direct voltages up to a maximum of approximately 30 volts. For operation on higher voltages, or where the normal low-voltage supply is subjected to high voltage pulses, transistors may, with suitable precautions, be connected in series.[47] This applies to both single-ended and push-pull inverters, and a typical single-ended circuit for operation on a 60-volt d.c. supply is shown in Fig. 11.

The base resistors, R_{B1} and R_{B2}, as well as adjusting the drive on the transistors, ensuring minimum collector-emitter voltage at the peak of the conducting half-cycle, also affect the relative sharing of the voltage appearing across the transistors in the non-conducting half-cycle. The transistors are bridged by voltage-dividing networks C_d and R_d, which will supply any out-of-balance transistor current.

The connection of the starter resistor, R_{S1}, directly to the collector of transistor V_1 instead of to the negative side of the supply ensures approximate sharing of the direct voltage should the inverter fail to start. The starting resistors also provide some degree of negative feedback, which allows a little wider tolerance in matching the transistor gains. Inverters of this type have been made to operate from direct voltages up to 140 volts.

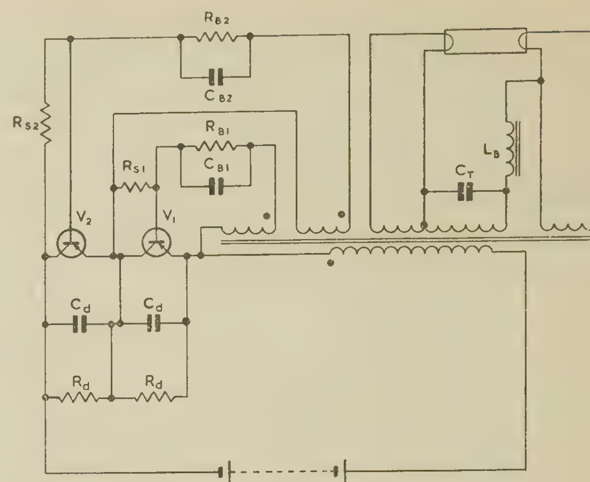

Fig. 11.—Class-C single-ended inverter with transistors in series.

(5.1.4) Split-Battery Inverter.

Another method of operating inverters on direct voltages greater than 30 volts is to use the 'split-battery' type of inverter,[48–50] of which a typical example is shown in Fig. 12.

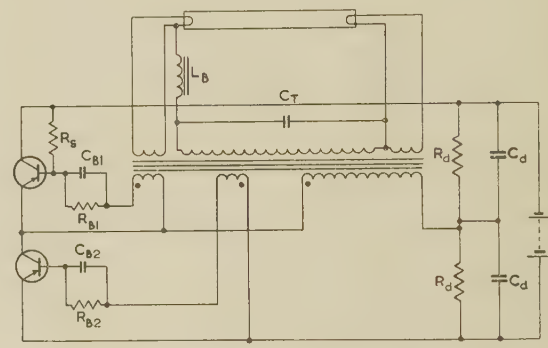

Fig. 12.—Push-pull split-battery inverter.

The transformer primary is connected between the junction of the two transistors, and either to a 'half-voltage' tapping on the battery or, as shown here, to the mid-point of a voltage-dividing network formed by C_d, R_d, etc.

The transistors have the same peak current and voltage stresses as in the series circuit, but the circuit has the advantages of push-pull operation and automatic sharing of the direct voltage between the transistors. Against this, only half of the available voltage is applied to the primary of the transformer, and consequently the step-up ratios are higher.

(5.1.5) Power Packs.

When inverters operate from direct voltages of 100 volts or more, the output powers become large and it is more economical to use the inverter as an a.c. power pack supplying several self-contained lamp circuits. Since these lamp circuits will normally be individually switched, it is preferable to use the split-battery circuit, since the push-pull nature of this circuit makes it more reliable in no-load conditions. The frequency of large inverters will probably be of the order of a few hundred cycles per second to enable lamp equipment already designed and in production for rotary inverters to be used.

(5.2) Square-Wave Circuits

When a square-wave inverter is used with a lamp having a reactive ballast, it is necessary to protect the transistors from

damaging voltage stresses. This can be achieved by connecting a capacitor across the primary of the transformer, but it has the disadvantage of increasing both the switching time of the transistors and the collector dissipation, resulting in an efficiency lower than is usually obtained in class-C sinusoidal inverters. In power-pack inverters special twin-lamp circuits can overcome these objections,[51] but this approach is uneconomic for small inverters. Nevertheless, the high efficiency of this type of inverter with resistive loads makes it very attractive. A circuit of this type used for lamp cathode preheating is described in the next Section.

For resistive loads the inverter may use any of the well-known circuits such as the push-pull, split-battery or bridge inverters.[52]

(5.2.1) Lamp-Cathode Pre-Heating Inverter.

Such a system is suitable for use in transport applications, when it is desired to operate several short fluorescent lamps from a 700 volt d.c. supply. In the circuit shown in Fig. 13,

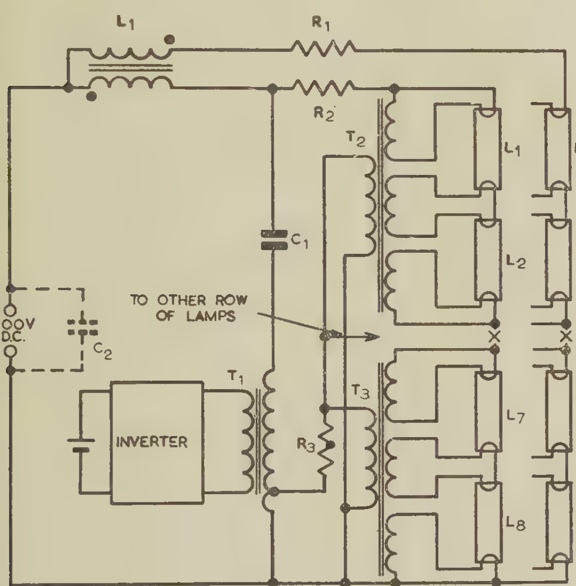

Fig. 13.—Lamp-cathode preheating and a.c. injection in a series d.c. system.

filament heating current for these lamps is supplied from a square-wave inverter operating from an auxiliary battery supply.[53] The same inverter also provides a high-voltage a.c. supply to ensure reliable lamp starting.

The circuit shows two groups each of eight lamps in series operating across the 700-volt d.c. supply with ballast resistors R_1 and R_2 in each arm. A winding on the inverter transformer, T_1, supplies 100 volts a.c., which is fed to the cathodes of the 16 lamps via step-down transformers T_2, etc. When cold, the tungsten cathodes have a low resistance and would constitute too heavy a load on the inverter and prevent it starting. To overcome this a negative-temperature-coefficient resistor, R_3, is included in the 100-volt line. The total secondary winding of the inverter transformer provides 1 kV a.c., which is injected into the 700-volt d.c. line via a small capacitor, C_1, to ensure reliable starting of the lamps. The choke L_1 virtually eliminates the alternating current flowing through the self-capacitance, C_2, of the d.c. supply; it also permits series a.c. injection into the second group of lamps.[54] The polarities of the two windings of this choke are as shown to keep the resultant d.c. flux in the core low when direct lamp currents are flowing in both windings.

(5.2.2) Future Developments in Square-Wave Inverters.

The most promising field for future developments of square-wave inverters for discharge-lamp loads appears to lie with the *p-n-p-n* switch.[55–57] Because of the thyratron-like properties of this device, the inverter circuit becomes substantially independent of the type of load, thus overcoming most of the objections to square-wave inverters with conventional *p-n-p* transistors. A typical inverter using these devices and having a blocking oscillator driving circuit is indicated in Fig. 14.

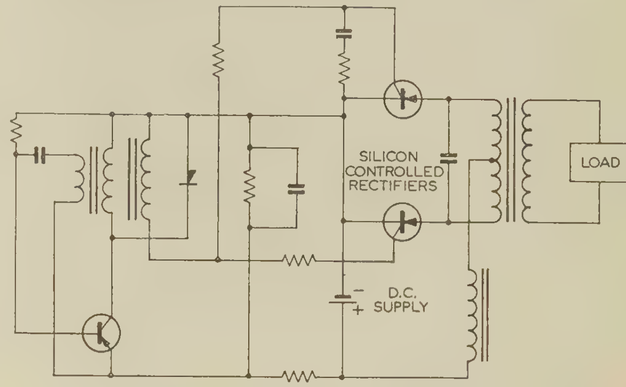

Fig. 14.—Driven square-wave inverter using *p-n-p-n* switches.

(6) OPERATION OF SEVERAL SINE-WAVE INVERTERS ON A COMMON D.C. SUPPLY

When a sine-wave inverter is designed to operate at a specific frequency the commercial tolerances of components will make a batch of inverters oscillate over a small range of frequencies about the design centre. If such a batch of inverters is connected to a common d.c. supply, one of three things may happen:

(*a*) If the d.c. supply impedance is sufficiently high, the oscillators will synchronize to a common frequency.
(*b*) If the supply impedance is sufficiently low, the inverters will oscillate independently at their own frequencies.
(*c*) Frequently the supply impedance is such that the oscillators beat with one another giving rise to visible flickering of the lamps.

The state of affairs in (*c*) may be overcome by connecting a suitable capacitor across the input of each inverter to lower the line impedance. A capacitor may be used by itself or as part of an *RC* filter network in the d.c. line. Alternatively, interconnected windings of a few turns on each transformer will synchronize all inverters to a common frequency.

(7) COMMERCIAL INSTALLATIONS

Transistor-inverter-operated fluorescent-lighting installations in the United Kingdom have been confined so far to transport applications, and most have used sine-wave inverters of one or other of the types described.

The first installation was in a number of the first-class coaches of the five trains used on the Euston–Glasgow 'Caledonian Express' of British Railways[58, 59]; 60 inverter fittings were installed, each operating a 20-watt fluorescent tube at 1 kc/s. The normal 24-volt d.c. coach supply is used. These inverters have now operated for 18 months, and both transistor and lamp performances have been entirely satisfactory.

Further large-scale railway coach installations[60] are in hand, and Fig. 15 shows a 40-watt sine-wave inverter which is to be installed in new British Railways and Pullman coaches. This inverter operates a 4 ft fluorescent tube and is mounted separately from the lamp and fitting.

Large numbers of sine-wave inverters are at present being

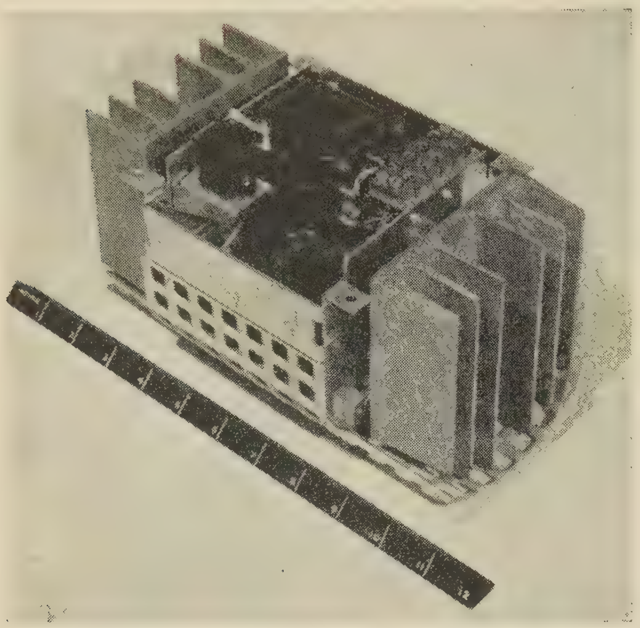

Fig. 15.—40-watt transistor inverter for British Railways.

installed in the Vickers Vanguard airliner,[61] to operate a 4 ft or 3 ft fluorescent tube at 15 kc/s from a 24-volt supply.

Other types of inverter which have been or are being developed and used range from a 12-volt d.c. unit for operating a 6-watt lamp in a car to a 110-volt d.c. power pack having a total lamp load of about 160 watts for use by British Railways.

This type of inverter has great possibilities both in large-scale installations as well as individual applications such as in caravans, boats and cars.

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DISCUSSION BEFORE THE UTILIZATION SECTION, 14TH JANUARY, 1960

Mr. L. J. Gardner: The development of transistors and their applications has proceeded rapidly over the past two years, but at the moment inverters of the type described are still limited by the inability of the single transistor to withstand supply voltages exceeding 30 or 40 volts. For higher voltages one must resort to series connection, but this is rather expensive and the controlled rectifier is likely to find a wider application here. Already these devices are available for operation up to 300 volts and 10 amp, and thus able to supply a considerable lighting load.

It is fairly reasonable to say that the development of transistor lighting units was started for railway lighting where, for various reasons, an inverter to operate a single lamp is preferable to a power pack to supply a number of lamps. With the higher-voltage supplies, however, this is not very economical and the power-pack type of inverter may become more prominent.

As an engineer I agree with the efficiency formula given in the paper, but when presenting the paper the authors showed a slide depicting the increase in the luminous efficiency of the lamp with frequency. It is fair that this should be included as part of the unit itself, although when discussing efficiency we must be careful to state whether we mean input/output power ratio, as in the paper, or light output per unit of d.c. power.

The paper refers briefly to noise, and the noise was appreciable during some of the demonstrations. The main difficulty with the noise from these units is not its amplitude but its very high frequency, which makes it very uncomfortable. One would normally consider that a railway carriage or a motor coach is a fairly noisy situation and that the noise from the lighting would not be very noticeable; unfortunately, because of the frequency it makes itself very apparent when one is exposed to it for long periods.

One other difficulty with transistors, which we have seen from the various models shown, is the trouble which must be taken to minimize the temperature of the junction. The designer can take a great deal of trouble to provide a heat sink, but the difficulty will arise when the user confines the units within very small fittings—which is usually desirable from some aspects but in this case is most undesirable. If these transistors find a widespread use, the user must be taught to provide adequate cooling and ventilation.

A paper of this type inevitably leads to speculation on possible future applications. The Americans have already done a lot of work on such transistor-inverter units for lighting public halls and for general industrial use, mainly because they reduce the size of the ballasting equipment and the associated losses. I think that the complications of this type of unit are such that it will be some time before the system, both in efficiency and in cost, becomes competitive with the standard methods which are available in this and other countries and on which quite a lot of work has already been done.

Mr. Th. Hehenkamp (*Netherlands*): My experiences roughly parallel those of the authors, although there are points of difference. We found that the square-wave inverter gave a very reliable performance, but was difficult to design with an efficiency higher than about 60%. The development of the sine-wave inverter has been a very big improvement, since it is easy to obtain an efficiency of 75–80% even with frequencies of 8–10 kc/s. It is true, as stated in the paper, that at high frequencies the load which can be switched by a certain type of transistor is reduced, but we found that the advantages of a higher lamp efficiency and a reduced noise level more than compensate for this. An extra advantage of a high frequency is the low weight of the inverter, and this becomes more important with the introduction of high-power units.

To-day these power packs can be made in the 100–200-watt range, but bigger units are still in the laboratory stage. As soon as this development has been completed, the low weight and the high efficiency can possibly lead to the introduction of transistorized equipment into the field of general lighting. In the next few years many interesting new developments can be expected.

Mr. K. A. E. Salmon: The authors have placed too much emphasis on the use of sine-wave inverters and made unfair criticism of the square-wave type. Very good results have been obtained using a compromise, where the main transformer primary is loaded with capacitance. The value required for optimum efficiency is governed by transient losses in the transistors, but a considerable gain in overall efficiency arises from improvement in the lamp-current waveforms. This circuit also permits the use of a high-reactance transformer, thus eliminating the choke, and makes possible compensated cathode heating as used in the normal switchless lamp circuit. Would the authors quote the efficiency they obtain from a push-pull sine-wave inverter and indicate their methods of measuring efficiency?

In controlled-rectifier inverters, I understand that some difficulty has been experienced in getting satisfactory output waveforms, owing to oscillation between the series inductance and the primary capacitance. Have the authors met this problem?

The authors refer to Campbell's paper on h.f. lighting using a triangular voltage supply; have they any further information on this?

Mr. G. M. Ward: For a considerable time my colleague and I have been concerned with convertors in one way and another, although not specifically concerned with fluorescent lamps, and I had hoped that the authors would discuss the economics of the present situation. The economics of transistorized sources of light must be considered relative to those of normal filament lamps, and I should like a few fundamental facts.

How do the rates of failure of filament and fluorescent lamps compare? What is the inherent cost of replacement and the relative overall efficiency? What is the relationship between the d.c. input power and the light output? In convertors we have to consider the overall power loss in the convertor plus the ballast loss in the choke. Despite the fact that luminous efficiency increases with higher operating frequency, surely the

increased loss in convertors due to saturation-resistance limitation and hole-storage effects may be a dominant consideration with British types.

When travelling by Underground I notice that with the 50-volt filament lamps a considerable number of short-duration surges occur; one moment the lamps are almost out and the next moment they are at full brilliance. What effect would this have with fluorescent lighting? It is almost an impracticable suggestion to consider using the devices discussed in the paper, for the flickering of fluorescent lamps would be a physiological nuisance.

Mr. S. Anderson: The first point on which I should like to comment is the use of the term 'split battery', for the battery is not split in the illustrations. Transport engineers would strongly object to splitting their batteries to make these 50-volt circuits operate. A better term might be a 'voltage-divider system', which is obviously applicable without any alteration to the battery.

Several allusions have been made to the important question of efficiency, but nothing has been said about methods of measuring it, and it is not an easy thing to measure without special equipment. Quite apart from this difficulty, I feel that we should agree on what we mean by the efficiency of one of these devices. I suggest that we should disregard differences in the luminous efficiency of the lamp which occur at different supply frequencies, for I object to confusing the issue by including an effect which, fortunate though it may be, is not attributable to the transistor inverter. I suggest that, for the purpose of an engineering appraisal, efficiency should be quoted in terms of the ratio between the power coming from the inverter and that supplied to it.

Mr. J. M. Waddell: In Section 4.6 the authors examine the thermal rating of transistors. They say that the maximum junction temperature at which the transistor may be operated safely is specified by the manufacturer, and then discuss methods of making use of this information. However, the device manufacturers are often one or two years ahead of the users, and thus I believe that most transistor designers to-day would prefer not to use the concept of maximum junction temperature as a method of rating, but to use a curve relating the power rating with the temperature of a point on the heat sink just below the transistor. This change in method of rating arises from a greater understanding of the behaviour of transistors on the part of manufacturers.

I disagree with Mr. Anderson on the question of efficiency. I suggest that what the customer (as an engineer) is interested in is the comparison between the light output and the power input (from the battery). If the manufacturers of rotary machines can use a frequency as high as 15 kc/s, they, too, will be able to claim a similar efficiency. If they cannot, then I suggest that some advantage is to be gained directly from the use of the transistor inverter.

I support the remarks about *p–n–p–n* controlled rectifiers. Apart from being of silicon, so that cooling is no great problem, they do not suffer from variation of current gain with increasing collector current. This enables heavy-current devices to be made much more easily than is the case with transistors. Not only shall we see the controlled rectifier used in a year or two at very high working voltages, but we shall also see it at very heavy currents—and it is worth noting here that there is a difference in the definition of rated current used by rectifier and transistor engineers. An entry of 10 amp on a data sheet for a controlled rectifier means average current, whereas on a power transistor data sheet it means peak current. There is thus a ratio of power-handling capacity of at least 2 : 1 in a square-wave circuit and probably more than that in most circuits.

In my experience, the use of *p–n–p–n* controlled rectifiers in an inverter does not inherently imply a square-wave type operation. With suitable circuit constants it is possible to obtain a sine wave. It is, however, in all cases necessary for stability to have a leading-power-factor load, and for efficiency the power factor should be near unity. These requirements can make inverter design for varying loads a little difficult, but with discrete lamp loads the necessary correction capacitors could be incorporated in each lamp circuit.

The authors refer to surges. Nowadays there are high-power Zener diodes available in this country, and these provide a convenient way of limiting, not only transient surges, but also long-term over-voltages.

Mr. N. Herwald: Several speakers have referred to the question of efficiency, and I agree that the efficiency must be clearly defined when fluorescent lamps are fed from inverters. A lamp operating in the normal manner from a 50 c/s source is rated at 40 watts when it takes 40 watts from the supply. It then gives a certain luminous output. The same tube operating from an inverter, at a much higher frequency and giving the same luminous output, will not take 40 watts from the inverter output. If the efficiency is calculated on the basis of the rating of the lamp being the actual output load, one is bound to find a higher calculated efficiency than if the actual power output were used in the ratio between output and input power. I suggest that this might be a case for an official definition.

Various users, and even different manufacturers for aircraft, railways and buses, have their own specifications and regulations. Cannot standard specifications be agreed for these different classifications of use even at this early stage?

Mr. J. Tozer: When designing the lighting of a large building it is often necessary to provide an emergency lighting system. In these days the normal lighting is often provided by fluorescent lamps, and because of difficulties with direct current we normally have to install additional tungsten lamps for emergency lighting. Do the authors think that with these modern developments we shall be able to install a complete fluorescent-lamp scheme and just switch in the inverters on the failure of a.c. supply?

THE AUTHORS' REPLY TO THE ABOVE DISCUSSION

Messrs. I. F. Davies and **D. Dunthorne** (*in reply*): Several speakers raise the question of the definition of efficiency as applied to a transistor-inverter-operated lamp circuit. We agree with Mr. Anderson that the only definition, from an engineering point of view, is that given in Section 3.2, i.e. the ratio of total power in the lamp to total d.c. input power. Since this definition is based on electrical parameters only, it gives the same circuit efficiency for a given size and rating of lamp regardless of the light output—which would not be the case if the efficiency was taken to be light output per unit of d.c. power, as suggested by Messrs. Gardner and Waddell. Typical efficiencies of sine-wave inverters measured in this manner are between 65 and 70%; these are lower than those mentioned by Mr. Hehenkamp, but we presume his efficiency measurements include the increase in lamp luminous efficiency occurring at higher frequencies.

As regards the measurement of efficiency, the only difficulty arises in the measurement of total lamp power (arc power plus cathode power). This can be carried out either by conventional methods using high-frequency calibrated wattmeters or by the restoration of light method, which is described in Reference 62 of the paper, and has been adopted in the draft for the second edition of I.E.C. Publication 82, 'Recommendations for Ballasts

for Fluorescent Lamps'. The measurements made by the latter method must be carried out at the same frequency.

Adequate cooling of the transistors is of paramount importance in these inverters, and we agree with Mr. Gardner that users of this type of equipment must be educated in the necessity for providing this facility.

Mr. Salmon prefers the saturating-transformer inverter with capacitively loaded primary, and suggests this is superior to the sine-wave type. In our experience, however, transformers of this type are inherently noisy, and efficiencies exceeding 60% are difficult to obtain with this system. The provision of compensated cathode heating is possible using a sine-wave inverter, owing to the disparity in load between the starting and running conditions of the lamp.

Our fears regarding the reliability of the modified square-wave inverter have been justified fully by the high failure rate of this type of inverter on the St. Pancras–Bedford line of British Railways; on the other hand, many thousands of sine-wave inverters have given faultless service over periods as long as two years.

In controlled rectifier inverters unwanted oscillations can occur between the series inductor and the commutating capacitor. The values of these two components must be matched carefully to the load if instability is to be avoided.

We have no experience of voltage supplies having a triangular waveform.

We cannot agree with Mr. Ward that the economics of transistorized sources of light must be considered ıelative to those of normal filament lamps. Compared with the filament lamp the fluorescent lamp has a longer life, higher efficiency, better colour appearance and colour rendering and has resulted in the great improvement in the lighting of buildings during the last two decades. The public now expects an improvement in the lighting levels in public transport, and the transport undertakings must realize that the lighting installation must carry a higher proportion of the capital cost of the vehicle than heretofore.

The rated average life of a 40-watt fluorescent lamp, which costs 13s. 9d., is 5000 hours and it has an average efficiency through life of between 42 and 65 lumens/watt depending on the colour; in comparison, a 40-watt 24-volt transport-type filament lamp has a life of 1000 hours, costs 3s. 6d. and has an efficiency of 13 lumens/watt.

We agree that in inverters, using the types of power transistor at present available, there will be an optimum frequency of operation (approximately 6kc/s) above which the increased transistor losses will cancel any gains due to the increase in lamp luminous efficiency with frequency. In practice, the operating frequency chosen will be a compromise between efficiency, size and cost.

The surges on London Underground trains, due to track interruptions, will have no effect on transistor-inverter-operated fluorescent lamps, since these are operated from a battery which is charged from the track voltage via a rotary generator.

In reply to Mr. Waddell we must say that, as circuit designers, we can work only from information supplied by the device manufacturers, and thus we welcome the trend to specify the transistor power rating in terms of the temperature of a point on the heat sink just below the transistor.

Mr. Herwald's suggestion that the transport operators should agree on a specification for this type of equipment is admirable; however, we are not very optimistic that this will come about, bearing in mind that many large users of lighting equipment are not prepared to accept even a British Standard without the addition of clauses specified by themselves.

The use of transistorized emergency lighting, as suggested by Mr. Tozer, is an idea which we feel will certainly be considered in the future.

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THE APPLICATION OF LINEAR INDUCTION MOTORS TO CONVEYORS

By E. R. LAITHWAITE, M.Sc., Ph.D., Associate Member, D. TIPPING, B.Sc., and D. E. HESMONDHALGH, M.Sc.

(*The paper was first received* 30*th September, and in revised form* 10*th December*, 1959.)

SUMMARY

The principal advantage in using linear induction motors to drive conveyor belts is that force can be applied uniformly to the belt over a wide area without mechanical contact. The drive is therefore independent of the coefficient of friction between belt and rollers and belt stretch is less likely to occur. The main problem in designing a linear motor for such a drive arises from the fact that the speeds required are low, and it is shown that efficient systems are possible only if the motor is supplied with low-frequency power.

Two systems are investigated, the first using a woven copper belt and the second a series of solid plates connected to chains along each side. The effect of end-ring resistance and of contact resistance between weft and warp is investigated in the case of the woven belt. The action of the plate conveyor involves the behaviour of discontinuous rotors, and a theoretical investigation of this problem is included. The findings are substantiated by experimental results obtained from a fairly large model. Other applications which could utilize the short rotor effect are suggested.

LIST OF PRINCIPAL SYMBOLS

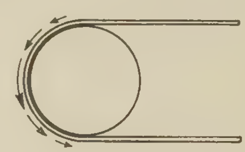
a = Rotor length.
b = Instantaneous flux density.
B = Peak flux density at a point.
B_0 = Peak flux density at leading edge of rotor.
B_a = Peak flux density at trailing edge of rotor.
c = End-ring width.
d = Stator slot depth.
f = Supply frequency.
F = Force.
F_D = Mean force.
g = Air-gap length.
I_s = Stator current.
j_r = Instantaneous rotor surface current density.
j_s = Instantaneous stator surface current density.
J_s = Peak stator surface current density.
k = Slot width/pitch ratio.
K = Overall belt width.
n = Number of poles on a stator block.
p = Pole pitch.
P_o = Power output.
R_1 = Stator resistance.
R_2 = Rotor resistance.
R_m = Resistive component of magnetizing impedance.
s = Distance measured from leading edge of rotor.
t_s = Time spent under stator block when travelling at field velocity.
u = Rotor speed.
u_s = Synchronous speed.
w = Stator width.
X_1 = Stator leakage reactance.
X_2 = Rotor leakage reactance.
X_m = Magnetizing reactance.
Z_1 = Stator impedance.
Z_2 = Effective rotor impedance.
Z_m = Magnetizing impedance.
η = Efficiency.
μ_0 = Permeability of free space.
ρ_c = Resistivity of copper.
ρ_r = Rotor surface resistivity.
ρ_s = Stator surface resistivity.
ρ_{r0} = Rotor surface resistivity neglecting end-ring resistance.
ρ_{s0} = Stator surface resistivity neglecting end-ring resistance.
σ = Fractional slip.
τ = Rotor time-constant.
ω = Angular frequency of supply.

Written contributions on papers published without being read at meetings are invited for consideration with a view to publication.

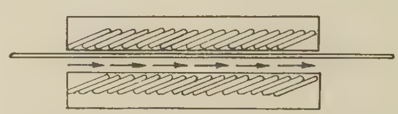
Dr. Laithwaite and Mr. Tipping are, and Mr. Hesmondhalgh was formerly, in the Electrical Engineering Department, University of Manchester. Mr. Hesmondhalgh is now in the Electrical Engineering Department, Queen's University, Belfast.

(1) INTRODUCTION

During discussions which followed the publication of a recent paper on linear induction motors[1] it was suggested that linear motors might be applied with advantage to the driving of certain types of conveyor. In particular, coal conveyors, which are normally operated through rollers driven by conventional induction motors, have the disadvantage that the force is applied to the belt over a comparatively small area, as shown in Fig. 1(*a*),



(*a*)



(*b*)

Fig. 1.—Application of driving force to a moving belt.
(*a*) Conventional roller drive.
(*b*) Linear-motor drive.

and the large forces imparted to the belt as it encounters a drive roller are liable to stretch it. If stretching occurs, the belt is liable to slip, so that some of the drive is lost and excessive wear occurs. Conditions of working may range from very dry and dusty to very wet and slimy, and it is difficult to make roller-driven conveyors operate satisfactorily under all conditions of load. If, for example, the power is applied to the belt at more than one point, the driving units are effectively interconnected only by means of the belt, which is elastic. Hunting, unequal loading and excessive slip are almost certain to occur.

The proposal to utilize linear induction motors involves the use of a belt of conducting material which passes between a pair of linear stator blocks carrying a polyphase system of coils connected so as to drive a travelling magnetic field through the

belt, the latter constituting the 'rotor' of the induction machine. Such a system, as illustrated in Fig. 1(*b*), would exert a uniformly distributed force over a large area of belt without mechanical contact: the drive is therefore independent of the coefficient of friction. A reasonably large experimental machine of this type has been constructed and tested; the theory of operation has been verified from the results obtained, and it is now possible to assess the possibilities of linear motors as conveyor drives.

The paper describes two possible forms of flexible belt, the first consisting of woven copper wire incorporating stainless steel wire for mechanical strength, and the second type made up from conducting plates carried on chains. The theoretical predictions of the behaviour of such a system involves the study of induction machines with discontinuous rotors—a subject about which little has been written.

(2) REQUIREMENTS OF CONVEYORS

There are many types of conveyor system, ranging from coal transporters and production-line belts to passenger conveyors such as escalators and lifts, but most types have one feature in common, namely that the linear speed rarely exceeds 10 ft/sec. Generally the conveyor takes the form of an endless belt, with only a portion carrying the load and the remainder comprising the return path.

The proposal to use linear motors as driving mechanism demands that the drive shall take place over sections of the belt which are not carrying goods, so that the belt can pass between the stators with the minimum clearance. The disadvantage of single-sided linear systems is that the magnetic circuit of the machine is extremely poor unless iron of sufficient core depth to carry the flux is incorporated in the moving member. It is not proposed to use conveyor belts containing iron, first because the belts would be thick and unmanageable and secondly because magnetic attraction between the rotor and the stator would introduce serious mechanical problems.

Fig. 2(*a*) illustrates how double-sided linear motors may be included in the active side of a conveyor system as well as in the return path, provided that the goods can be dropped through a small distance from one part of the belt to another. Fig. 2(*b*) shows an alternative system which is more economical as regards total length of belt: this system could possibly be developed for systems where dropping of the goods was impossible by the use of small fixed regions, as shown in Fig. 2(*c*).

(3) PROPERTIES OF LINEAR INDUCTION MOTORS

The quality of a motor may be assessed differently for different applications: some users place the emphasis on high efficiency and power factor, while first cost is almost always important. But however the quality is judged, a parameter which is most potent in deciding the 'goodness' of a machine is the surface speed of the moving conductors. The maximum force per unit area which can be exerted across the air-gap of a machine is fixed by the limits of the electric and magnetic circuits, and these limits vary surprisingly little for wide ranges of machine size, speed and supply frequency.

The maximum power output obtainable from a given frame size with a given cooling system is therefore approximately proportional to the surface speed.

Few conventional rotary induction machines are manufactured in sizes greater than 1 h.p. in which the surface speed is less than 20 ft/sec. In the design of conventional machines use is made of the equivalent circuit of the machine, as shown in Fig. 3, in which the imperfections of the electric and magnetic circuits are represented by series and shunt impedances, Z_1, Z_2 and Z_m.

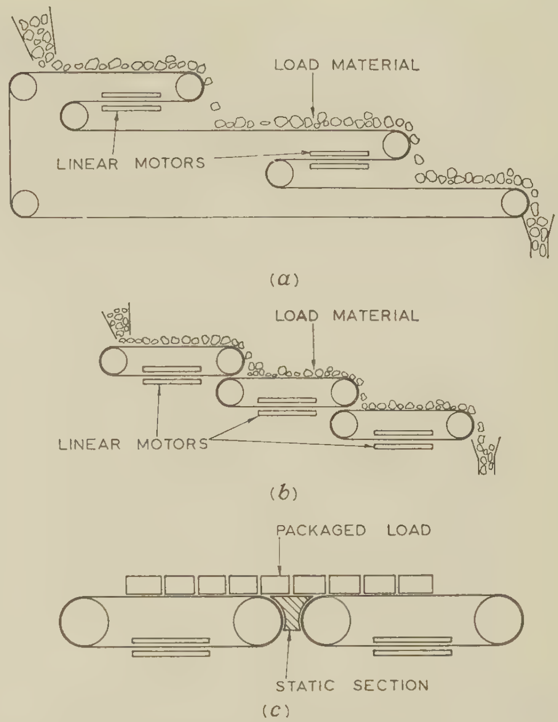


Fig. 2.—Conveyor systems using linear-motor drives.

(*a*) Continuous-belt system.
(*b*) Short-belt system.
(*c*) Static-section system.

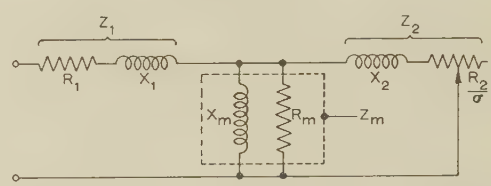


Fig. 3.—Equivalent circuit of the induction motor.

For most well-designed rotary machines it can be assumed that Z_m is very high in relation to Z_1 and Z_2; indeed it is usually much larger than R_2/σ, where σ is the working slip. In such cases most of the operating characteristics of the machine can be predicted from open- and short-circuit tests, and it can easily be shown that the efficiency is very low if R_1 and R_2 equal or exceed Z_m.

The magnetizing impedance, Z_m, is often largely reactive and R_m can always be made higher by the use of thinner laminations; X_m is proportional to the area of one pole, which for a given core length is proportional to the pole pitch, p. Rotor and stator resistances are dependent on the amount of conductor contained within a pole pitch. For the space available around the periphery and for a tolerable amount of leakage, the amount of conductor is proportional to p, neglecting end turns, so that R_1 and R_2 are proportional to $1/p$. The rotor time-constant, $\tau = L_m/R_2$ (where $L_m = X_m/2\pi f$) is therefore proportional to p^2. Only if $\tau\omega \gg 1$ can the normal approximations for the prediction of performance be used. The performance of motors for which $\tau\omega$ is less than 10 would not be acceptable for most conventional rotary machines of reasonably large size unless $R_1 \ll R_2$.

The value of τ in terms of machine dimensions has been shown to be given by[2]

$$\tau = \frac{4p^2\mu_0}{\pi\rho_r g} \qquad (1)$$

where ρ_r is the equivalent rotor surface resistivity, g is the air-gap length and μ_0 is the permeability of free space. Eqn. (1) takes no account of rotor leakage reactance or of end-ring impedance.

As an example of the method of calculating ρ_r, a squirrel-cage rotor in which the slot depth is d and the slot width is equal to k times the slot pitch has a value of ρ_r given by ρ_c/kd, where ρ_c is the resistivity of the conducting material in the slots, assuming the latter to be completely filled.

In linear motors in which the rotor contains no iron all the rotor conductor lies in the air-gap, so that if the gap could be completely filled with conductor and end-ring resistance could be neglected, the product $\rho_r g$ would be constant and equal to ρ_c. The maximum value of $\tau\omega$ for such a machine which has a copper rotor and is fed from a 50 c/s supply is therefore $0{\cdot}188p^2$ (assuming $\rho_c = 2{\cdot}13 \times 10^{-6}$). If $\tau\omega$ is to exceed 10, p must be greater then 2·9 in and the linear synchronous speed must be greater than 24 ft/sec. In practice, some mechanical clearance is required between rotor and stator and end-turn resistance may be considerable. The minimum values of ρ_r and of synchronous speed are therefore likely to be considerably higher than these values for the condition $\tau\omega > 10$.

The low-speed requirement of conveyor systems involves low values of $\tau\omega$ for 50 c/s supply, which in turn demands very low stator resistances if high efficiency is to be maintained. Stator resistance can be reduced only by increasing the slot depth, which increases both leakage and cost.

Considerable improvement may be obtained at the expense of supplying the motor at low frequency. For a given surface synchronous speed pf is constant, whereas $\tau\omega$, being proportional to p^2f, increases linearly with p. The upper limit to p occurs when end-turn length becomes comparable with active length. For a low-frequency supply the iron-loss component of the no-load current is likely to be small in comparison with the magnetizing current, which is likely to be higher than for most conventional machines. Since the rotor conductor is not contained in slots and the frequency is low, rotor leakage reactance may be neglected in comparison with rotor resistance. The output, P_0, of a machine whose stator current is I_s may be calculated from the equivalent circuit in Fig. 3, making the above assumptions, and is given by

$$P_0 = I_s^2 R_1 \left\{ \frac{1-\sigma}{\dfrac{R_1}{R_2}\left[\sigma^2 + \left(\dfrac{1}{\tau\omega}\right)^2\right]} \right\} \quad . \quad . \quad . \quad (2)$$

while the efficiency, η, is given by

$$\eta = \frac{1-\sigma}{1 + \dfrac{1}{\sigma}\left\{\dfrac{R_1}{R_2}\left[\sigma^2 + \left(\dfrac{1}{\tau\omega}\right)^2\right]\right\}} \quad . \quad . \quad . \quad (3)$$

Fig. 4 shows the theoretical variation of maximum efficiency with R_1/R_2 for various values of $\tau\omega$.

Rotor copper loss is not likely to limit the power output of conveyor systems, since the rotor conductor spends only a fraction of its time under a stator block, the greater fraction being spent in open air where natural cooling is very effective. Eqn. (2) expresses the power output in terms of stator copper loss, which is likely to set the limit on output. Further investigation on the practically realizable values of P_0 and η cannot be attempted until the nature of the belt is known, so that R_2 and $\tau\omega$ can be assessed.

(3.1) Short-Stator Effect

Eqns. (2) and (3) assume that the stator of the linear motor is infinitely long, so that the equivalent circuit shown in Fig. 3 applies.

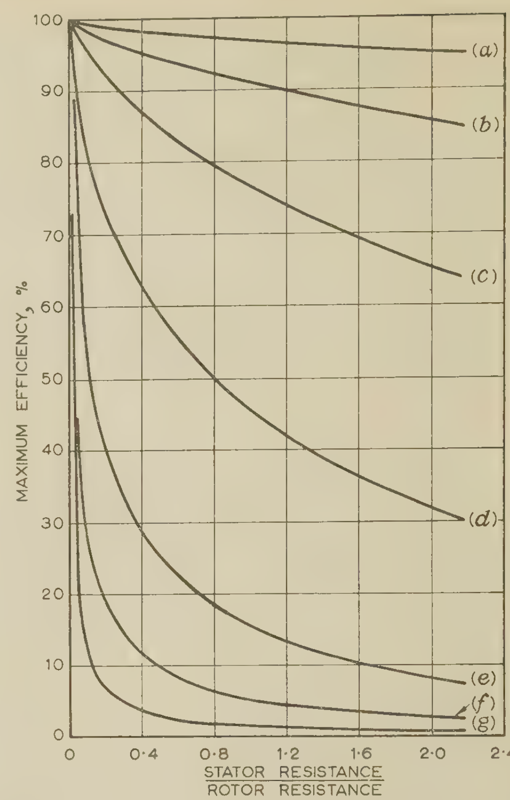


Fig. 4.—Effects of R_1/R_2 and $\tau\omega$ on maximum efficiency.

(a) $\tau\omega = 100$.
(b) $\tau\omega = 30$.
(c) $\tau\omega = 10$.
(d) $\tau\omega = 3$.
(e) $\tau\omega = 1$.
(f) $\tau\omega = \frac{1}{2}$.
(g) $\tau\omega = \frac{1}{4}$

An essential difference between the operation of linear and rotary induction machines occurs as the result of the transient phenomena associated with the entry and exit edges of the stator blocks in the linear case. The theory of the operation of short-stator machines has been formulated in some detail,[2–4] and the salient features may be summarized as follows:

(a) Transient rotor currents are set up at the entry edge. Parallel connection of the stator windings without end-grading is inadvisable, since the resulting currents in the stator coils at the entry edge are certain to be excessive. With series connection the flux density just inside the entry edge of the stator is very low.

(b) The transient rotor current set up in any rotor bar decays with rotor time-constant as the bar proceeds under the stator. So long as transient rotor current is present it modulates the amplitude of the flux wave in the air-gap.

(c) This modulation of the flux can result in extra losses not calculable by more conventional theory. The extra loss has been shown to be zero if the machine runs at slip values of $1/(n+1)$, $2/(n+2)$, etc., where n is the number of poles on a block and the rotor time-constant is so large that the rotor transient has not decayed appreciably by the time the rotor bars emerge.

(d) At the exit edge similar transient rotor currents are set up which result in rotor copper loss occurring outside the stator block, so introducing further excess loss. Such losses have been shown to be zero for slip values of $2/(n+2)$, $4/(n+4)$, etc., with the same provision as before.

(e) If the rotor time-constant is small or the number of poles very large, or the slip is large, the rotor transient currents become insignificant by comparison with steady-state current before the rotor bars emerge and thereafter the motor performs conventionally.

For a linear motor used for a conveyor it is clear that the rotor time-constant is likely to be much smaller than that of a rotary machine. It is considered that, if the time, t_s, spent by a

rotor bar in traversing the entire stator length at synchronous speed is greater than 5τ, the corrections to conventional theory introduced by short-stator effect are negligible. For values of $t_s/\tau < 5$ it is advisable to calculate the performance on the short-stator theory.[3]

(3.2) Woven-Wire Belts

One type of flexible belt designed and tested consisted of woven copper wire. Weaving wire is not an easy process, and the difficulty increases with the size of wire used. An attempt was made to simulate a squirrel-cage rotor in that comparatively thick copper wires were woven together with stranded thin wire to act as end-rings; the wire which had to bend as the belt rounded a drum was thus thin and flexible, and it was hoped that during the bending process the thick wires would rotate within the loops of the stranded wire in the manner of hinges, thereby maintaining good electrical contact. A belt consisting entirely of copper is liable to stretch easily, and so some steel wire was introduced to strengthen the belt longitudinally; non-magnetic stainless steel was used so that it could be incorporated in the active centre section, leaving the overhung sections completely free to be filled with copper to obtain a low end-ring resistance.

The amount of copper which can be included in the air-gap with woven wire belts is restricted by the nature of the weave. The overall thickness is greater than the diameter of the active wires by twice the diameter of the steel wire. The space between active wires is about equal to their diameter for wires of about 0·1 in diameter and relatively greater for thicker ones. A factor of $\pi/4$ is lost by the use of circular wire. The first belt used on the experimental machine had an overall belt thickness of 0·146 in, but its resistivity was equal to that of copper sheet only 0·021 in thick.

If the contact resistance between weft and warp is zero, the resistance of the end-rings for the type of belt described may be calculated by a method due to Russell and Norsworthy.[5] This is completed in Section 9.1 and the results are shown in Fig. 5. Clearly there is little gain in increasing end-ring width beyond

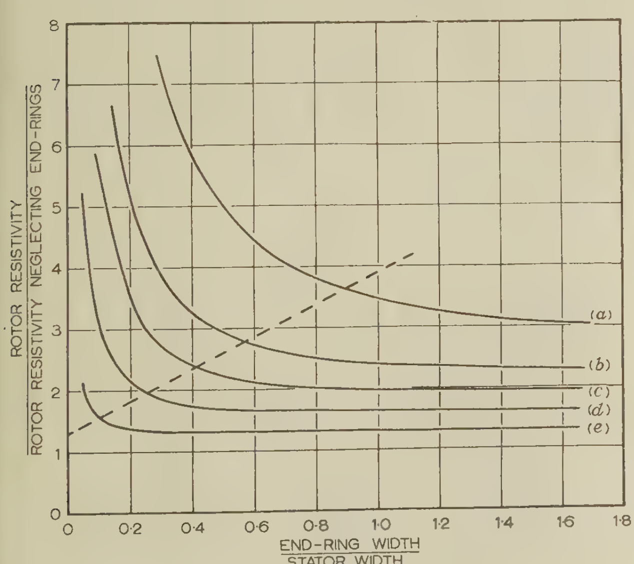


Fig. 5.—Effective increase in rotor resistivity of a woven belt due to end-rings.

(a) $p/w = 3$.
(b) $p/w = 2$.
(c) $p/w = 1\frac{1}{2}$.
(d) $p/w = 1$.
(e) $p/w = \frac{1}{2}$.

a certain point. For example, the designer may decide that an economic end-ring width would be one in which the effective resistance was, say, 20% greater than the value for an infinitely wide end-ring. If such a criterion is adopted the locus of points corresponding to the 20% increase is shown by the broken line in Fig. 5, and it can be seen that for all practical purposes it may be regarded as a straight line which is divided uniformly by the constant p/w curves, so that equations relating end-ring resistance, p/w and c/w may be written as follows:*

$$\frac{\rho_r}{\rho_{r0}} = 1\cdot17 + 0\cdot78\frac{p}{w} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and

$$\frac{c}{w} = -0\cdot05 + 0\cdot3\frac{p}{w} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where ρ_{r0} is the resistivity of the active rotor conductor only.

The effective stator resistivity for the active length, ρ_{s0}, is inversely proportional to the stator slot depth, to the width/pitch ratio of the slot and to the packing factor in a slot. Stator end-turn may be taken into account once the form of coil has been fixed. For example, the stator resistivity for a particular type of diamond coil may be given by the following empirical formula, if w is in inches:

$$\frac{\rho_s}{\rho_{s0}} = 1 + 1\cdot41\frac{p}{w} + \frac{2}{w} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Eqns. (1), (4), (5) and (6), eqn. (3) with ρ_s/ρ_r substituted for R_1/R_2 and the following two fundamental equations enable the efficiency of a linear conveyor belt to be predicted:

Belt speed, $\quad u = 2pf(1 - \sigma)$ . . . . (7)

Overall belt width, $\quad K = w + 2c$ . . . . (8)

In general, u and K will be fixed by the application and some restriction will be placed on ρ_{s0} by considerations of cost and power factor.

(4) AN EXPERIMENTAL MACHINE

An experimental machine was built to verify as much of the foregoing theory as possible. Problems of handling restricted the width of the stator to 6 in, and a 9 in pole pitch was chosen as being reasonably large without introducing excessive end-ring resistance. Although eqn. (5) indicates that an end-ring width of only 2·4 in is required for 20% increase in resistivity, there was considerable doubt about the contact resistance between warp and weft of the woven belt, and the end-rings were made 6 in wide to ensure low resistivity. Stator blocks were made 10 ft long, which was thought sufficient to ensure that short-stator effects could be neglected. The stator slot width was half the slot pitch and the slot depth was 1·5 in. The stator resistivity was measured and found to be $8\cdot23 \times 10^{-6}$ ohm-cm, which is $4\cdot62\rho_c$.

As stated earlier, the resistivity of the active part of the woven belt was approximately seven times that of a copper sheet of the same overall thickness, so that the value of ρ_r, including end-rings and assuming perfect contact between weft and warp, was about $37\cdot5\rho_c$ (see Fig. 5). The belt was supported at its edges and it was found necessary to use an air-gap of 0·44 in to ensure that the belt wires did not foul the stator teeth. No attempt was made to improve the mechanical arrangement to reduce the air-gap, since it was felt that the effect of the large gap was readily calculable. The calculated value of $\tau\omega$ at 10 c/s was therefore 0·55. The properties of this belt when run at 7·5 ft/sec

* Eqns. (4) and (5) refer to the 20% increase; any other percentage increase would introduce different constants.

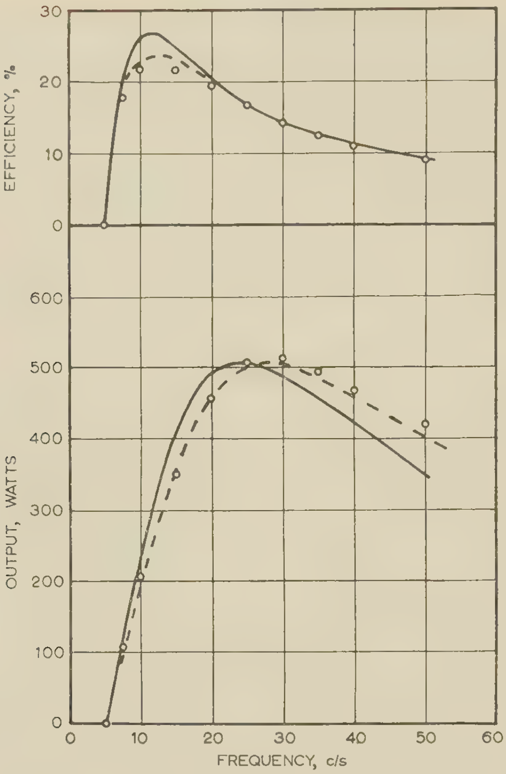


Fig. 6.—Efficiency and output of woven belt conveyor at 7·5 ft/sec.
—— Theoretical curves assuming no weft/warp contact resistance.
– – – – Theoretical curves assuming 25% increase due to contact resistance.
○ ○ ○ Experimental points.

were investigated. Fig. 6 shows the experimental results for efficiency and output compared with the curves predicted by eqns. (3) and (4), assuming no contact resistance between weft and warp (full lines) and a 25% increase in resistivity due to this effect (broken lines).

These results suggest that there is a slight increase in ρ_r due to contact resistance. It is of interest to calculate the maximum efficiency which could have been achieved with this belt and pair of stator blocks. For example, had the air-gap been reduced to 0·184 in, the peak efficiency would have been 41·5% with a supply frequency of 8·33 c/s. A still higher efficiency would be obtained if the pole pitch were increased. An optimum efficiency of 58% is obtained at 6·25 c/s with a pole pitch of 18 in and the same stator slot depth, but any further increase of pole pitch for the 6 in-wide stator is detrimental.

For the best results with woven belts, the wire of the belt should be sufficiently rigid to support itself between the stator blocks without reinforcing wire, so that ρ_r can be minimized. If necessary, reinforcing wire can be included in the end-rings. The following data form an example of the high efficiency obtainable with a larger belt:

Overall belt width	36 in
Thickness of copper bars	$\frac{1}{4}$ in
Spacing of bars	1 bar every $\frac{3}{4}$ in
Stator width	30 in
End-ring width	3 in
Pole pitch	10 in
Air-gap	0·45 in
Stator slot depth	3 in
Overall efficiency at a belt speed of 7·5 ft/sec	61% (with a supply frequency of 5 c/s)

The power output at peak efficiency from such a machine would be of the order of 3·2 watts per square centimetre of stator surface for a stator copper loss of 1 watt/cm^2. For the 30 in-wide stator this is equivalent to about 10 h.p. per foot length.

(5) PLATE CONVEYORS

Flexibility may be achieved without recourse to weaving wire by constructing the belt in the form of a series of metal plates which are bolted to two chains positioned along the edges of the belt. The chains run on sprockets which replace the rollers of the previous system. The use of solid plates appears advantageous from the aspect of obtaining a low value of ρ_r, and it is true that values of $\rho_r g$ approaching ρ_c are possible, provided that the individual plates are long compared with a pole pitch of the stator, so that the induction-motor action could be said to be conventional, and provided that the motor is wide enough in relation to a pole pitch for end-turn losses to be negligible. As is often the case in design, however, there are two conflicting requirements: the maximum length of the plates is limited mechanically by the required degree of flexibility, involving the clearances required when an individual plate is carried around a sprocket; at the same time, the shorter the plates, the greater is the restriction on the flow of rotor current, and purely electrical considerations are liable to place a restriction on the effectiveness of plate conveyors.

The behaviour of induction machines whose rotor consists of discontinuous sections of conductor, in particular when the sections are two pole pitches or less in length, is not well known, although some theoretical work has been carried out by Shturman.[6, 7]

(5.1) Short-Rotor Effect

The action has many features which are similar to those of a short-stator machine, for the behaviour of both types is largely dependent on the method of connection of the stator: *parallel* connection of short-*stator* machines results in high *current* density *inside* the edge of the stator, unless end-grading is used; *series* connection of short-*rotor* machines, on the other hand, results in high *flux* density *outside* the edges of the rotor. The principle of duality may be used to predict the properties of one from those of the other with the flux density and current density interchanging roles between short-stator and short-rotor effects, although it must be remembered that, when short rotors are used in sets, the behaviour will depend to a large extent on the space between adjacent conducting sections. It should also be remembered that the short-rotor counterpart

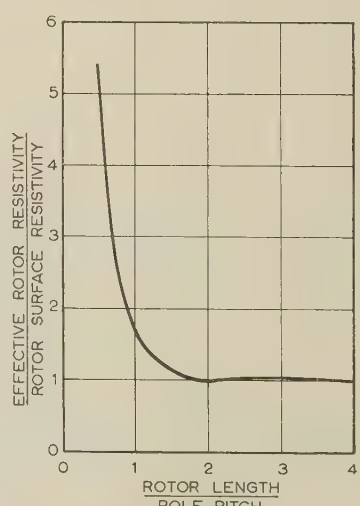


Fig. 7.—Effective rotor resistivity of a constant-flux short-rotor machine.

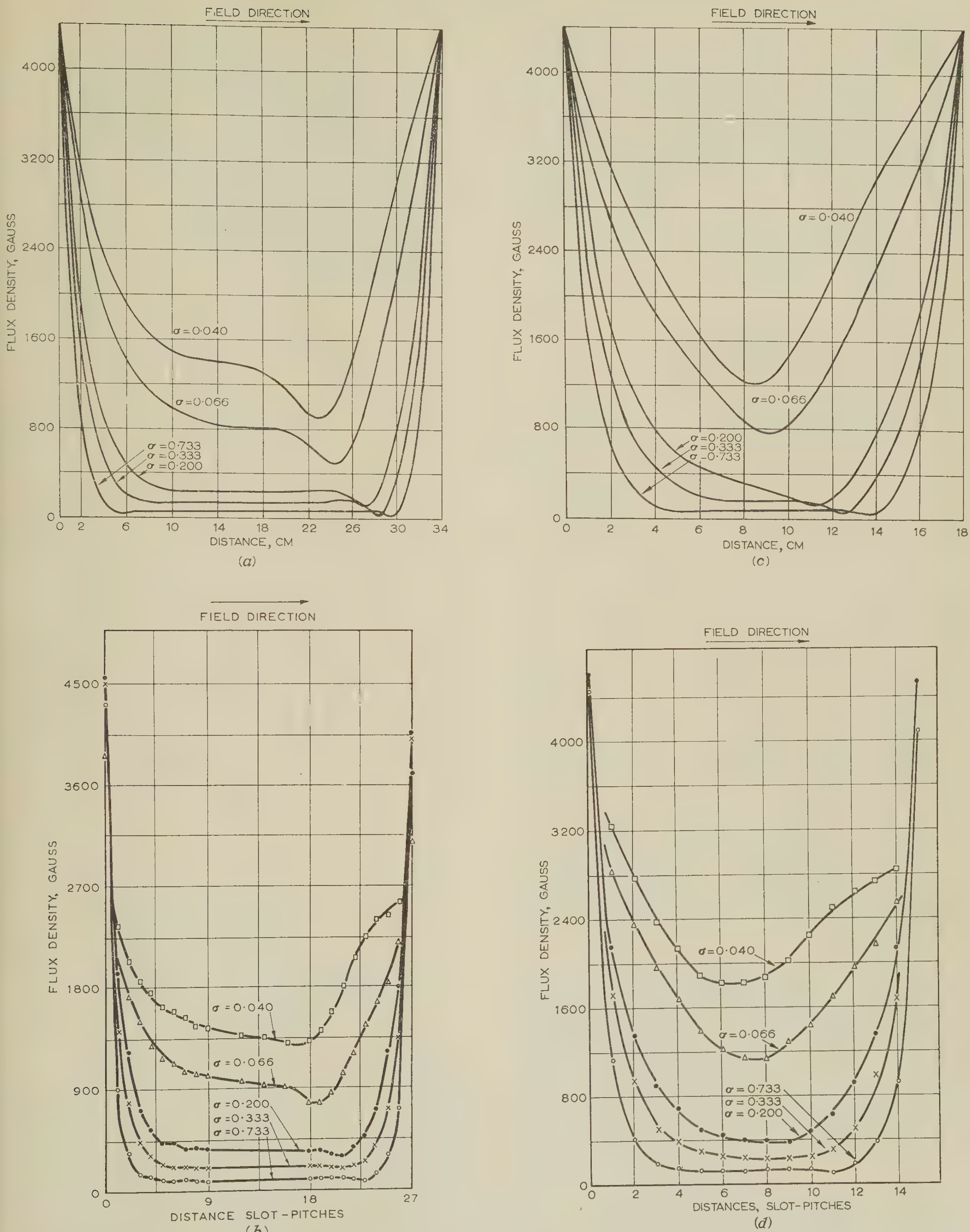


Fig. 8.—Flux distributions across a short rotor.

(*a*) Computed curves for $a = 33\cdot3$ cm, $a/p = 2$.
(*b*) Experimental results for $a = 33\cdot3$ cm, $a/p = 2$.
(*c*) Computed curves for $a = 17\cdot91$ cm, $a/p = 1$.
(*d*) Experimental results for $a = 17\cdot91$ cm, $a/p = 1$.

of a short-stator machine would contain an iron-cored rotor, the iron extending just as far as the conductor, and such a system is clearly different from one in which the rotor iron is continued indefinitely on either side of the short conducting section so as to make the magnetic circuit identical, both inside and outside the rotor. The double-sided motor, whose rotor consists of conductor only, falls into this latter category.

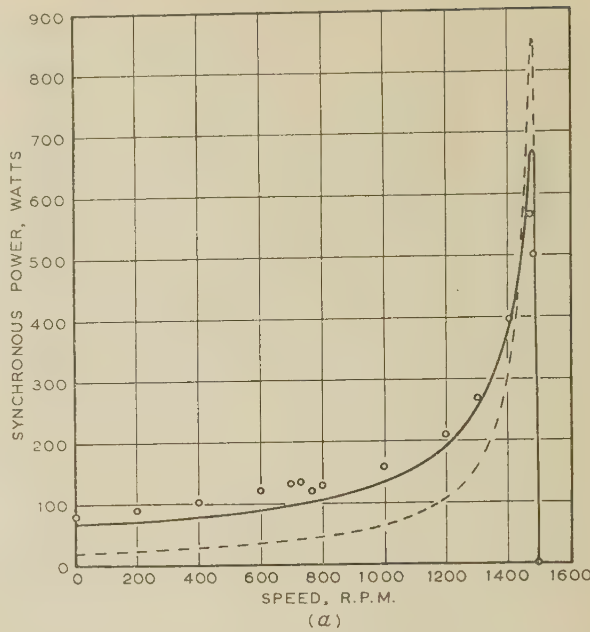
The current distribution impressed and the force exerted on a short rotor by a parallel-connected stator which provides a uniform travelling wave of flux are evaluated in Section 9.2. Shturman[6, 7] analysed a similar system, including the effect of a high-resistance end-ring connecting adjacent short-rotor sections, such as may be obtained in practice if a cast rotor has saw cuts made in the end-rings. The analysis of Section 9.2 is in agreement with that due to Shturman if the joining end-ring impedance is infinite. The rotor is seen to behave like that of a conventional motor with a higher resistivity than would be calculated on conventional theory from its dimensions. The effective resistivity, ρ_r, is plotted against rotor length in Fig. 7.

The theory of the transient behaviour of a single short rotor operated on by a series-connected stator is developed in Section 9.3. Since very few induction machines are parallel-connected, in the sense that the flux in every tooth is separately forced, the constant-current theory is likely to be more generally applicable to short-rotor machines. As with short-stator machines, the transient phenomena are not easily distinguished in machines with a low value of $\tau\omega$, and the theory of Section 9.3 was verified using a conventional rotary-machine stator with a 4-pole winding and a rotor in which only one short grid of rotor bars was inserted.

Figs. 8(*a*) and (*c*) show the theoretical flux distributions for rotors of two and one pole pitches in length, while Figs. 8(*b*) and (*d*) show the measured flux distributions using a series of search coils on the rotor connected to a voltmeter through slip-rings. Further evidence of agreement between theory and practice is shown in Fig. 9, in which speed/torque curves are compared.

The practical machine displayed a half-speed crawling phenomenon which was not predicted by the theory of Section 9.3. The theoretical expression for j_r may be obtained by substituting for j_s and b from eqn. (27) in eqn. (17). The resulting expression contains terms which represent waves starting from both $s = 0$ and $s = a$ travelling in opposite directions across the rotor with exponentially decaying amplitude. Under these conditions there can be sets of standing waves set up which themselves have exponentially decaying amplitudes, and examination of the form of these standing-wave components reveals that in general their sum is not zero so that the rotor current as a whole will have a pulsating component.

Should the stator not be perfectly current-fed, sub-harmonic currents will flow in the stator winding and give rise to half-speed crawling torques according to the well-known Görges effect.[8] Half-speed crawling torques, observed with the experimental machine are absent from the theoretical curves, owing to the assumption of perfect stator current feed. It was verified experimentally that a better current feed, produced by inserting impedance in series with the stator, reduced the relative size of the half-speed torque component. The crawling phenomenon is almost certainly due to Görges effect.



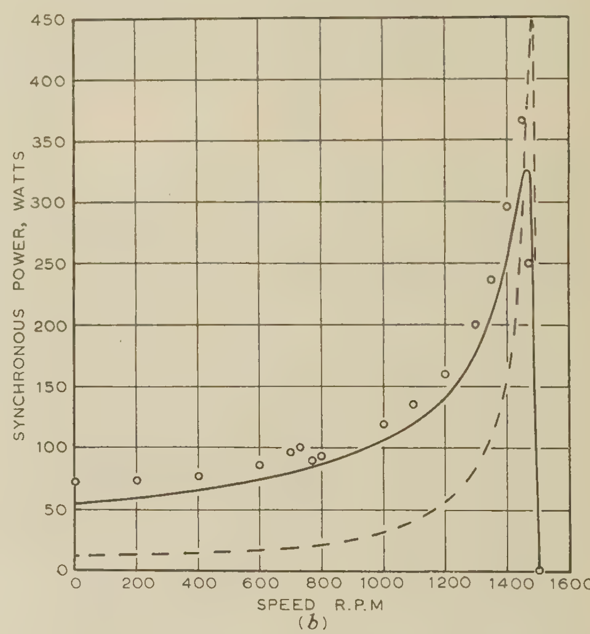


Fig. 9.—Speed/torque curves of short-rotor machines.
——— Calculated curves for short-rotor machine.
– – – – Calculated curves for equivalent conventional machine.
○○○ Experimental points.
(*a*) $a/p = 2$.
(*b*) $a/p = 1$.

(5.2) Experiments with Plate Conveyors

Tests were carried out on the same linear stator as that described in Section 4, the structure being modified to enable the stator to drive a plate belt consisting of sheets of aluminium ¼ in thick and 18 in wide. Belts with plates of length 3, 6, 9 and 18 in were tested, the method of mounting the plates being shown in Fig. 10. Brake tests for a range of frequencies were carried out with each of the different rotor lengths, the belt speed in each case being maintained at 7·5 ft/sec. Some of the results are shown in Fig. 11; the calculated curves show remarkable agreement, considering that no exact calculation exists for the effect of end-ring resistance in a short rotor. The calculated curves of Fig. 11 are based on eqn. (29), together with an end-ring resistance as calculated by Russell and Norsworthy[5] for a continuous sheet. It is clear that, in spite of the short-rotor effect, higher rotor conductivities and hence higher efficiencies are possible when using plates than when using a woven belt

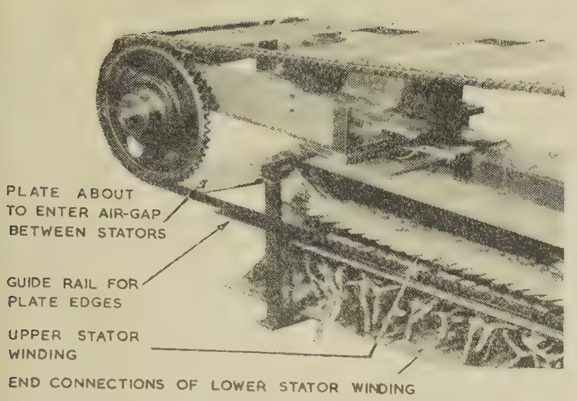


Fig. 10.—The experimental plate conveyor.

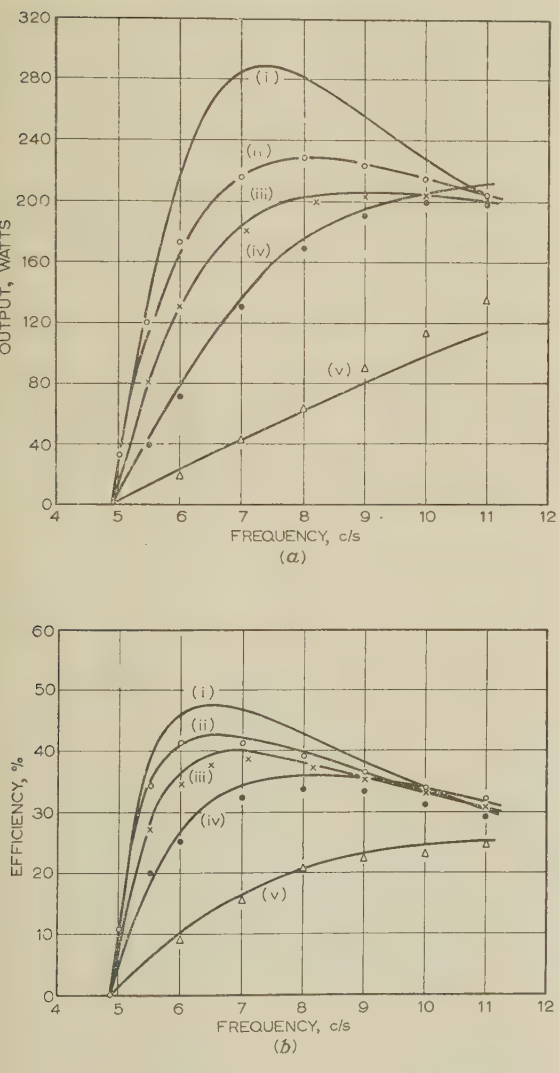


Fig. 11.—Output and efficiency of a plate conveyor.

Calculated curves for:
(i) Equivalent conventional machine.
(ii) $a/p = 2$. ○ ○ Experimental results.
(iii) $a/p = 1$. × × Experimental results.
(iv) $a/p = 2/3$. ● ● Experimental results.
(v) $a/p = 1/3$. △ △ Experimental results.
(*a*) Output curves.
(*b*) Efficiency curves.

for the same size structure, even for plates only one-third of a pole pitch in length. When comparing the two sets of results from the experimental machine it must be remembered that the conductivity of the plate system is relevant to aluminium, whose resistivity is some 1·6 times that of copper.

Eqn. (29) may be used to plot curves from which the performance of a given design of linear motor may then be predicted. Eqn. (4) is replaced by a combination of eqn. (29) and the expression for end-ring resistance of a homogeneous sheet,[5] namely

$$\frac{\rho_r}{\rho_{r0}} = \frac{1}{1 - \dfrac{\tanh \dfrac{\pi w}{2p}}{\dfrac{\pi w}{2p}(1+\gamma)}}$$

where $$\gamma = \tanh \frac{\pi w}{2p} \tanh \frac{\pi c}{p}$$

The design outlined in Section 4 may be compared with that of a plate conveyor of the same size supplied at the same frequency, the design data being as follows:

Stator width	30 in
Thickness of plates	¼ in
Plate width	36 in
Plate length	10 in
Pole pitch	10 in
Air-gap	0·45 in
Stator slot depth	3 in
Overall efficiency at 7·5 ft/sec (5 c/s supply)	75%
Power output at peak efficiency	4·5 watts per watt of stator copper loss

It is also instructive to calculate the increase in supply frequency which is possible using a plate conveyor with the same stator as above in order to achieve the same efficiency as that of the woven belt. For such a design the pole pitch could be reduced to 5 in and the frequency increased to 12 c/s, when the power output would be approximately equal to that of the woven belt.

(6) CONCLUSIONS

Most of the experimental work was carried out at a belt speed of 7·5 ft/sec, the choice of speed being quite arbitrary. It is clear that linear machines with properties almost as good as those of rotary machines can be built for surface speeds comparable with those of the latter, but since it was felt that in this application the main problem was that of the low-speed requirement, attention was focused on this aspect. The main disadvantage of using linear induction motors for conveyors is that they require a low-frequency supply, the cost of which is probably prohibitive for many applications. Apart from this, it is clear that machines can be designed to have high efficiency and power factor and reasonable power output.

It should be noted that the efficiencies and power output quoted for the designs in Sections 4 and 5.2 are not the optimum values for belt speeds of 7·5 ft/sec and that still higher figures are possible at lower frequencies. The designer must, however, take into consideration the overall performance of the linear motor and frequency-changer.

The short-rotor effect discussed in the second half of the paper has possible applications in other fields. Short-stator machines are designed to have a particular rotor resistivity which dictates the flux density in the machine. Some of the recent developments of brushless variable-speed motors,[9, 10] involve the use of short-stator machines with adjustable pole-pitches. It may be

possible to use short-rotor effect to advantage in such devices in order to shape the output characteristics, since a rotor two pole-pitches long for the low-speed setting becomes effectively one pole-pitch long at double speed, with corresponding increase in effective resistivity and hence of output. A detailed consideration of this application is, however, beyond the scope of the present paper.

Shturman[6, 7] hoped to use the effective increase in resistivity of a short rotor to improve the starting properties of squirrel-cage induction motors, and it is true that an 'inverted' machine, in which the primary winding constitutes the rotor, which is fed through slip rings, can make use of a variety of stator connections which effectively change the a/p ratio to improve the starting torque or even to produce variable-speed running.

(7) ACKNOWLEDGMENTS

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(9) APPENDICES

(9.1) End-Turn Resistance

The basic equations from which the flow lines of current in the end-rings of induction motors and hence the effective end-ring resistance can be determined are given in Reference 5. Evaluation of this resistance involves the substitution of four surface resistivities, two of which refer to the active conductor in two directions along and perpendicular to the field velocity and the other two to the end-ring conductor in the same directions.

For the woven belt, three of these resistivities may be regarded as equal, the other being the resistivity under the stator in the axis containing the velocity vector. This fourth resistivity may be regarded as infinite. In this case the equations of Russell and Norsworthy may be used to evaluate the ratio of the total effective resistivity, ρ_r, to the resistivity ρ_{r0} which would exist if the end-rings were of zero resistance and

$$\frac{\rho_r}{\rho_{r0}} = 1 + \frac{2p}{\pi w}\coth\frac{\pi c}{p} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

where c is the width of the overhang, so that the total belt width is $w + 2c$.

Eqn. (9) is represented in Fig. 5 in which ρ_r/ρ_{r0} is plotted against c/w for various values of p/w.

(9.2) Current Distribution and Force Developed in a Rotor acted on by a Constant-Amplitude Flux Wave

The short rotor shown in Fig. 12 is considered to be infinitely thin with a surface resistivity ρ_r. If the relative velocity between

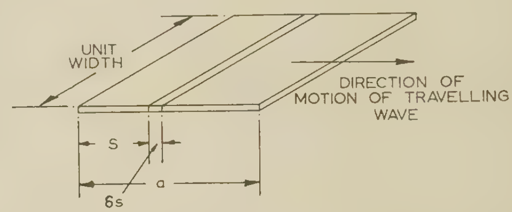


Fig. 12.—Convention used in the analysis of Section 9.2.

the travelling wave and the rotor is σu_s the e.m.f., v, induced in an elementary strip distant s from one edge is given by

$$v = b\sigma u_s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where the flux density b is given by

$$b = B\sin\left(\frac{\pi s}{p} - \sigma\omega t\right) \quad . \quad . \quad . \quad . \quad (11)$$

It is assumed that the rotor under consideration is of unit length at right angles to the direction of motion, being part of a much wider rotor in which end-turn impedance can be neglected. The leakage flux due to the rotor conductor is also neglected.

If the element considered is the only source of e.m.f., a current $j_r ds$ flows along the element, j_r being the rotor current density, and returns as a uniform current density across the area $a - \delta s$, where the return current density is $j_r/(a - \delta s)$, which tends to j_r/a as δs tends to zero. In the limit, where the rotor is considered to be divided into an infinite number of elements, the current density in the element which is at distance s from the edge is given by current due to its own e.m.f. acting through its own resistance, less the return currents from all other elements Thus:

$$j_r = \frac{v}{\rho_r} - \frac{1}{a}\int_0^a \frac{v}{\rho_r}ds$$

$$= \frac{B\sigma u_s}{\rho_r}\left[\sin\left(\frac{\pi s}{p} - \sigma\omega t\right) + \frac{2p}{\pi a}\cos\frac{\pi a}{2p}\cos\left(\frac{\pi a}{2p} - \sigma\omega t\right)\right] \quad . \quad . \quad . \quad . \quad (12)$$

which is a travelling wave of current density in which the base-line of the waveform shifts sinusoidally with time.

(9.2.1) **Force.**

The total force, F, on the rotor is given by

$$F = \int_0^a j_r b ds$$

Substituting for j_r from eqn. (12), b from eqn. (11) and integrating gives

$$F = \frac{B^2\sigma u_s}{\rho_r}\left[\frac{a}{2} - \frac{p}{2\pi}\sin\frac{\pi a}{2p}\cos\left(\frac{\pi a}{2p} - \sigma\omega t\right) + \frac{4p^2}{\pi^2 a}\sin^2\frac{\pi a}{2p}\sin^2\left(\frac{\pi a}{2p} - \sigma\omega t\right)\right] \quad . \quad (13)$$

The steady component, F_D, of F is therefore

$$F_D = \frac{B^2\sigma u_s a}{2\rho_r}\left(1 - \frac{4p^2}{\pi^2 a^2}\sin^2\frac{\pi a}{2p}\right) \quad . \quad . \quad . \quad (14)$$

or the force per unit area is

$$\frac{F_D}{A} = F'\left(1 - \frac{\sin^2\theta}{\theta^2}\right) \quad . \quad . \quad . \quad . \quad (15)$$

where F' is the force per unit area on a conventional rotor and $\theta = \pi a/2p$.

The effect of the rotor length may be considered as an increase of resistivity from ρ_r to $\rho_r/(1 - \sin^2\theta/\theta^2)$ and this effective resistivity is plotted against a/p in Fig. 7.

(9.3) **Flux Distribution and Force in a Short Rotor acted on by a Current-Fed Stator**

The stator and the rotor iron are assumed to be infinitely long, the stator and rotor currents are assumed to be concentrated in an infinitely thin layer of conductor on the surface, and the iron is assumed to be infinitely permeable and non-conducting and there is no component of H along the air-gap. Fig. 13

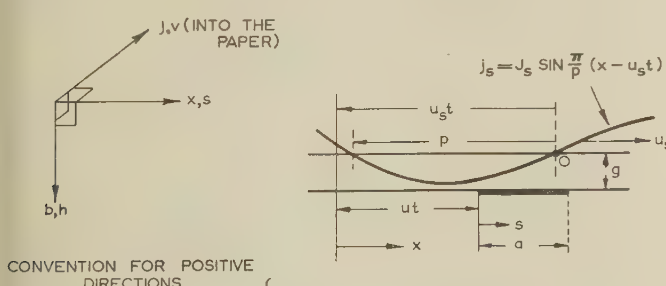


Fig. 13.—Convention used in the analysis of Section 9.3.

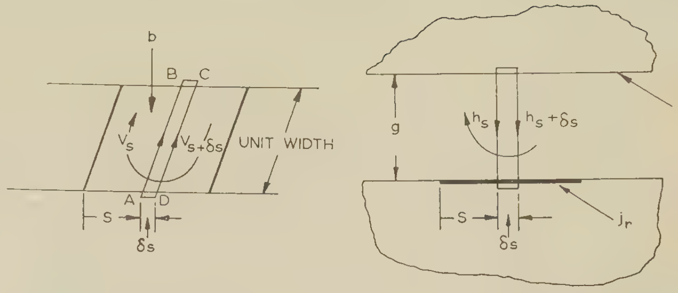
shows the relative positions of the stator wave and the rotor at time t, the origin of t being taken to be when the left-hand edge of the rotor is opposite the origin, O, on the current wave. The stator current density, j_s, is assumed to be

$$j_s = J_s \sin\frac{\pi}{p}(x - u_s t)$$

where x and p are as shown in Fig. 13. For points on the rotor distance s from one edge,

$$j_s = J_s \sin\frac{\pi}{p}[s - (u_s - u)t] = J_s \sin\left(\frac{\pi s}{p} - \sigma\omega t\right)$$

j_s may therefore be written as $\mathscr{I} J_s \exp j\left(\frac{\pi s}{p} - \sigma\omega t\right)$.



(a) (b)

Fig. 14.—Elementary loops used in the derivation of eqns. (16) and (17).

The calculations are based on unit width of rotor, as shown in Fig. 14(a). For the elementary loop ABCD,

$$\rho_r \frac{\partial j_r}{\partial s} = \frac{\partial b}{\partial t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

For the elementary path shown in Fig. 14(b),

$$j_s + j_r = g\frac{\partial H}{\partial s} = \frac{g}{\mu_0}\frac{\partial b}{\partial s} \quad . \quad . \quad . \quad . \quad (17)$$

From eqns. (16) and (17),

$$\frac{g}{\mu_0}\frac{\partial^2 b}{\partial s^2} = \frac{\partial j_s}{\partial s} + \frac{1}{\rho_r}\frac{\partial b}{\partial t} \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Now, a fixed point on the rotor will experience a sinusoidal variation of b at slip frequency in the absence of return current, so that b may be written

$$b = B\varepsilon^{-j\sigma\omega t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

where B is a function only of s. Substitution of this expression and the one for j_s in eqn. (18) yields

$$\frac{g}{\mu_0}\frac{d^2 B}{ds^2} + \frac{j\sigma\omega B}{\rho_r} = \frac{j\pi}{p}J_s \exp\frac{j\pi s}{p} \quad . \quad . \quad . \quad (20)$$

Writing $\frac{\sigma\pi^2}{p^2}\tau\omega = \lambda$, the complete solution of eqn. (20) is

$$B = C_0 \exp\frac{j\pi s}{p} + C_1 \exp js\sqrt{(j\lambda)} + C_2 \exp - js\sqrt{(j\lambda)} \quad . \quad (21)$$

where
$$C_0 = \frac{\frac{\rho_r J_s}{u_s}\left(\sigma - \frac{j}{\tau\omega}\right)}{\sigma^2 + \left(\frac{1}{\tau\omega}\right)^2} = \alpha + j\beta, \text{ say} \quad . \quad . \quad (22)$$

As in the short-stator theory, it is assumed that the flux density is a continuous function across the boundaries, so that

at $s = 0$, $\qquad b = B_0 = B_1 + jB_2$ (say)

and at $s = a$, $\qquad b = B_a = B_3 + jB_4$

where B_0 and B_a are the flux densities immediately outside the edges of the rotor.

Thus
$$B_0 = C_0 + C_1 + C_2 \quad . \quad . \quad . \quad . \quad (23)$$

and
$$B_a = C_0 \exp\frac{j\pi a}{p} + C_1 \exp ja\sqrt{(j\lambda)} + C_2 \exp - ja\sqrt{(j\lambda)} \quad . \quad . \quad . \quad . \quad (24)$$

Solving eqns. (23) and (24) for C_1 and C_2,

$$C_1 = \frac{C_0 \exp \frac{j\pi a}{p} - B_a + (B_0 - C_0) \exp - ja\sqrt{(j\lambda)}}{\exp - ja\sqrt{(j\lambda)} - \exp ja\sqrt{(j\lambda)}} \quad . \quad (25)$$

$$C_2 = \frac{B_a - C_0 \exp \frac{j\pi a}{p} - (B_0 - C_0) \exp ja\sqrt{(j\lambda)}}{\exp - ja\sqrt{(j\lambda)} - \exp ja\sqrt{(j\lambda)}} \quad . \quad (26)$$

Substitution of the values of C_0, C_1 and C_2 from eqns. (22), (25) and (26) in eqns. (19) and (21) gives the general expression for the flux density b. Since $j_s = J_s \sin(\pi s/p - \sigma\omega t)$, $B_1 = 0$ and $B_2 = -(4p\mu_0/g)J_s$ (see Reference 2). B_2 may be written

$$B_2 = -\frac{\rho_r J_s \tau\omega}{u_s}$$

$$B_3 = \frac{\rho_r J_s \tau\omega}{u_s} \sin \frac{\pi a}{p}$$

and

$$B_4 = -\frac{\rho_r J_s \tau\omega}{u_s} \cos \frac{\pi a}{p}$$

Substituting the values of these constants into the general expression for the flux density and simplifying,

$$b = \frac{J_s \rho_r}{u_s}\left[\frac{\sigma}{\sigma^2 + \left(\frac{1}{\tau\omega}\right)^2}\right]\left[\sin\left(\frac{\pi s}{p} - \sigma\omega t\right)\right.$$

$$- \frac{1}{\sigma\tau\omega}\cos\left(\frac{\pi s}{p} - \sigma\omega t\right) + \frac{1}{2(\cosh 2qa - \cos 2qa)}$$

$$\left(\left\{\varepsilon^{q(a-s)} \sin\left[\frac{\pi a}{p} + q(a+s) - \sigma\omega t\right]\right.\right.$$

$$- \varepsilon^{-q(a+s)} \sin\left[\frac{\pi a}{p} - q(a-s) - \sigma\omega t\right]$$

$$- \varepsilon^{q(a+s)} \sin\left[\frac{\pi a}{p} + q(a-s) - \sigma\omega t\right]$$

$$\left. + \varepsilon^{-q(a-s)} \sin\left[\frac{\pi a}{p} - q(a+s) - \sigma\omega t\right]\right\}$$

$$+ \sigma\tau\omega\left\{\varepsilon^{q(a-s)} \cos\left[\frac{\pi a}{p} + q(a+s) - \sigma\omega t\right]\right.$$

$$- \varepsilon^{-q(a+s)} \cos\left[\frac{\pi a}{p} - q(a-s) - \sigma\omega t\right]$$

$$- \varepsilon^{q(a+s)} \cos\left[\frac{\pi a}{p} + q(a-s) - \sigma\omega t\right]$$

$$\left. + \varepsilon^{-q(a-s)} \cos\left[\frac{\pi a}{p} - q(a+s) - \sigma\omega t\right]\right\}$$

$$+ \left\{\varepsilon^{qs} \sin[q(2a-s) - \sigma\omega t] - \varepsilon^{-q(2a-s)} \sin(-qs - \sigma\omega t)\right.$$

$$\left. - \varepsilon^{q(2a-s)} \sin(qs - \sigma\omega t) + \varepsilon^{-qs} \sin[-q(2a-s) - \sigma\omega t]\right\}$$

$$+ \sigma\tau\omega\left\{\varepsilon^{qs} \cos[q(2a-s) - \sigma\omega t]\right.$$

$$- \varepsilon^{-q(2a-s)} \cos(-qs - \sigma\omega t) - \varepsilon^{q(2a-s)} \cos(qs - \sigma\omega t)$$

$$\left.\left.\left. + \varepsilon^{-qs} \cos[-q(2a-s) - \sigma\omega t]\right\}\right)\right] \quad . \quad . \quad . \quad . \quad (27)$$

where $q = \frac{\pi}{p}\sqrt{\frac{\sigma\tau\omega}{2}}$

This expression has been computed for two particular cases and the results are shown in Fig. 8.

(9.3.1) Force.

Writing $b = B_p \sin(\pi s/p - \sigma\omega t) + B_q \cos(\pi s/p - \sigma\omega t)$, where B_p and B_q are functions only of s, the total force F on the rotor is given by

$$F = \frac{J_s}{2}\int_0^a B_p ds \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

Evaluation of B_p from the final expression for flux density and integration gives the following expression for the force, where $m = pq/\pi = \sqrt{\tfrac{1}{2}\sigma\tau\omega}$,

$$F = \frac{\frac{1}{2}\rho_r J_s^2 a}{u_s}\left[\frac{\sigma}{\sigma^2 + \left(\frac{1}{\tau\omega}\right)^2}\right]\left(1 + \frac{1}{\frac{\pi a}{p}\left(\cosh\frac{2\pi am}{p} - \cos\frac{2\pi am}{p}\right)}\right.$$

$$\frac{1}{m^2 + (m-1)^2}$$

$$\left[(m-1)\sin\frac{2\pi am}{p} - 2(m-1)\sin\frac{\pi am}{p}\cos\frac{\pi a}{p}\cosh\frac{\pi am}{p}\right.$$

$$\left. + 2m\cos\frac{\pi am}{p}\cos\frac{\pi a}{p}\sinh\frac{\pi am}{p} - m\sinh\frac{2\pi am}{p}\right]$$

$$+ \frac{1}{m^2 + (m+1)^2}$$

$$\left[(m+1)\sin\frac{2\pi am}{p} - 2(m+1)\sin\frac{\pi am}{p}\cos\frac{\pi a}{p}\cosh\frac{\pi am}{p}\right.$$

$$\left. + 2m\cos\frac{\pi am}{p}\cos\frac{\pi a}{p}\sinh\frac{\pi am}{p} - m\sinh\frac{2\pi am}{p}\right]$$

$$+ \frac{\sigma\tau\omega}{m^2 + (m-1)^2}$$

$$\left[m\sin\frac{2\pi am}{p} - 2m\sin\frac{\pi am}{p}\cos\frac{\pi a}{p}\cosh\frac{\pi am}{p}\right.$$

$$\left. - 2(m-1)\cos\frac{\pi am}{p}\cos\frac{\pi a}{p}\sinh\frac{\pi am}{p} + (m-1)\sinh\frac{2\pi am}{p}\right]$$

$$+ \frac{\sigma\tau\omega}{m^2 + (m+1)^2}$$

$$\left[m\sin\frac{2\pi am}{p} - 2m\sin\frac{\pi am}{p}\cos\frac{\pi a}{p}\cosh\frac{\pi am}{p}\right.$$

$$\left.\left.\left. - 2(m+1)\cos\frac{\pi am}{p}\cos\frac{\pi a}{p}\sinh\frac{\pi am}{p} + (m+1)\sinh\frac{2\pi am}{p}\right]\right\}\right)$$

$$. \quad . \quad . \quad . \quad . \quad (29)$$

Eqn. (29) has been computed for two examples and the results are shown in Fig. 9. Other computed values using eqn. (29) are shown in Fig. 11.

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THE DESIGN OF HOUSING-ESTATE DISTRIBUTION SYSTEMS USING A DIGITAL COMPUTER

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SUMMARY

An automatic method for designing housing-estate distribution systems is described which offers a practical and economical alternative to present design techniques. Substation positions and feeding arrangements are calculated instead of being determined intuitively, resulting in better layouts and closer approaches to given engineering limits. The method was applied to four housing estates and achieved savings ranging from 6 to 15% of the m.v. mains expenditure required by the previous manual designs. Apart from capital savings, uniform standards of design are obtained and designers are relieved of much tedious work.

The automatic method can facilitate management decisions by supplying cost figures for variations in design. For example, alternative substation sites may be tested when optimum sites are difficult to secure, the cost of increase in assumed a.d.m.d. can be determined and variations in the number of standard cable sizes can be investigated.

Special features of the automatic method are the division of irregular estate areas and a road-selection procedure which determines the shortest route between two points.

(1) INTRODUCTION

The formulation of economic design practices for housing-estate distribution systems is of recent origin. There appears to have been little thought applied to this problem before the end of the last war, when the rise in the price of copper, together with restrictions on capital expenditure and extensive housing development, led to systematic investigations into the economics of supplying housing estates.[1,2] Based on an analysis of theoretical networks and statistical load investigations, a C.E.A. and Area Board report recommended considerable changes in design practice. Completely interconnected networks were shown to be economically unjustified, and, instead, radial distributors were recommended based on the single-main system (employing mains on one side of the road only), with cable cross-sections tapered towards the extremities. Formulae were evolved for the most economical number of substations and for the accurate calculation of cable sizes to cater for closely estimated future loads. This new design technique was adopted by most Area Boards and resulted in substantial capital savings.

However, the technique still requires much experience and skill. The designer cannot attempt to calculate the best substation positions and feeding routes because the time required would be prohibitive. Many decisions are therefore based on an intuitive examination of the estate layout.

The high speeds and logical facilities of a modern digital computing machine have been used in an automatic design system in which the computer calculates many alternative designs and selects the cheapest layout consistent with the given engineering criteria of maximum demand, voltage drop, cable rating and interconnection requirements. Even with the available high computing speeds the time required to calculate all possible combinations of substation positions and cable networks would still be prohibitive. The automatic design is therefore divided into a series of steps, thus substantially reducing the number of possibilities which need to be considered for the best design.

(2) DESIGN PROGRAMME*

The logical ideas on which the automatic design programme is based are described in detail in subsequent Sections. Briefly, the programme works as follows:

The computer calculates the number of substations. It then selects initial substation positions by an approximation procedure which ignores the road layout. The results may not be practicable; for example, a position may not be accessible. The automatic operation is therefore interrupted and the designer chooses several feasible sites in the vicinity of each initial position. The computer uses the sites nearest the initial positions to design an initial cable layout which is based on the shortest possible feeding routes. An improved cable layout follows which departs from the shortest-feed criterion where savings can be achieved, and this selects the cheapest combination of cable sizes for the substation sites used.

Other cable layouts are then tried with alternative sites in search of the best substation sites until the best overall design is selected by the machine.

For large estates the complete procedure is repeated with other numbers of substations, above and below the calculated number. Cable designs can be prepared for copper or aluminium conductors; alternatively, designs can be obtained for both types of cable so that the cheaper type can be selected.

(2.1) Initial Substation Positions

Fig. 1 shows the plan of one of four housing estates for which test designs were prepared.[3] In addition to the private dwellings there are several other classes of consumer; for each class an estimate of the ultimate after-diversity maximum demand must be made by the designer. In this particular case, the values for ultimate a.d.m.d. per consumer were taken as 2 kW for houses, 100 kW for schools, 3 kW for shops, 20 kW for nursery schools and 10 kW for the church and public house, respectively.

To simplify the calculation of initial substation positions, the estate area is divided into $\frac{1}{4}$-acre squares [Fig. 2(*a*)(i)], and the load within each unit square is assumed to be concentrated at the centre of that square. It is also assumed that the shortest possible straight-line distance connects each concentrated load to its supplying substation, i.e. the actual road routes are ignored. This simplifying assumption is reasonable for estates with well interconnected roads. Where roads are insufficiently interconnected for the electrical distribution, it was observed that the design engineer is almost invariably forced to route some cables off the road system; hence the simplifying assumption was also

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* A programme is a list of orders which, when supplied to the computer, will cause it to perform the desired operations automatically.

Fig. 1.—Development plan for estate No. 2.

Substation positions
Actual.
Calculated (optimum feasible site).
Coincident.
Estate boundary.

considered reasonable for such a case. The values of concentrated loads, together with x and y co-ordinates which identify the square, are supplied to the machine. This information effectively represents a load map of the estate for the calculation of the number and initial positions of substations. The number, n, of substations is computed using the formula[2]

$$n = \left(\frac{A \times C \times P}{S} \times 14 \cdot 51\right)^{1/2} \quad . \quad . \quad . \quad (1)$$

where A = Area of estate, acres.
C = Cost of copper for $0 \cdot 1$ in^2 4-core cable, £/yd.
P = Load of estate, kW.
S = That part of the substation cost which is independent of n.

The selection of substation positions is based on a minimum-cost criterion for voltage-drop restrictions only; transmission losses are not considered, because, at the present relative costs of copper and energy, the cost of losses may be neglected.[2]

Assuming full use to be made of the permitted voltage drop V, for each concentrated load I situated at a straight line distance l from its supplying substation the cross-sectional area or cable size a is given by

$$V = \rho\frac{Il}{a}$$

Hence $$a = K_1 Il$$

The cost, C, is approximately proportional to the total volum of conductor, so that, for the complete estate,

$$C = K_2 \Sigma Il^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (2$$

where K_1 and K_2 are constants and $l = (x^2 + y^2)^{1/2}$.
For minimum costs, eqn. (2) should be a minimum.

The calculation of initial substation positions presents tw problems:

(*a*) The allocation of concentrated loads to the nearest substatior
(*b*) The calculation of the best substation position within its are of supply.

An iterative process was evolved which starts with approximat substation positions. The concentrated loads are first allocate to the nearest substation, using the straight-line principl Improved substation positions are obtained by calculating th centre of gravity of load* for each area of supply and movin the substations from the approximate starting positions to th centres of gravity. The concentrated loads are then re-allocate to the nearest improved substation and new centres of gravit are calculated for the adjusted areas of supply. The process c re-allocation, centre-of-gravity calculation and movement to th

* From eqn. (2) it follows that minimum costs are achieved when the substatic coincides with the centre of gravity.

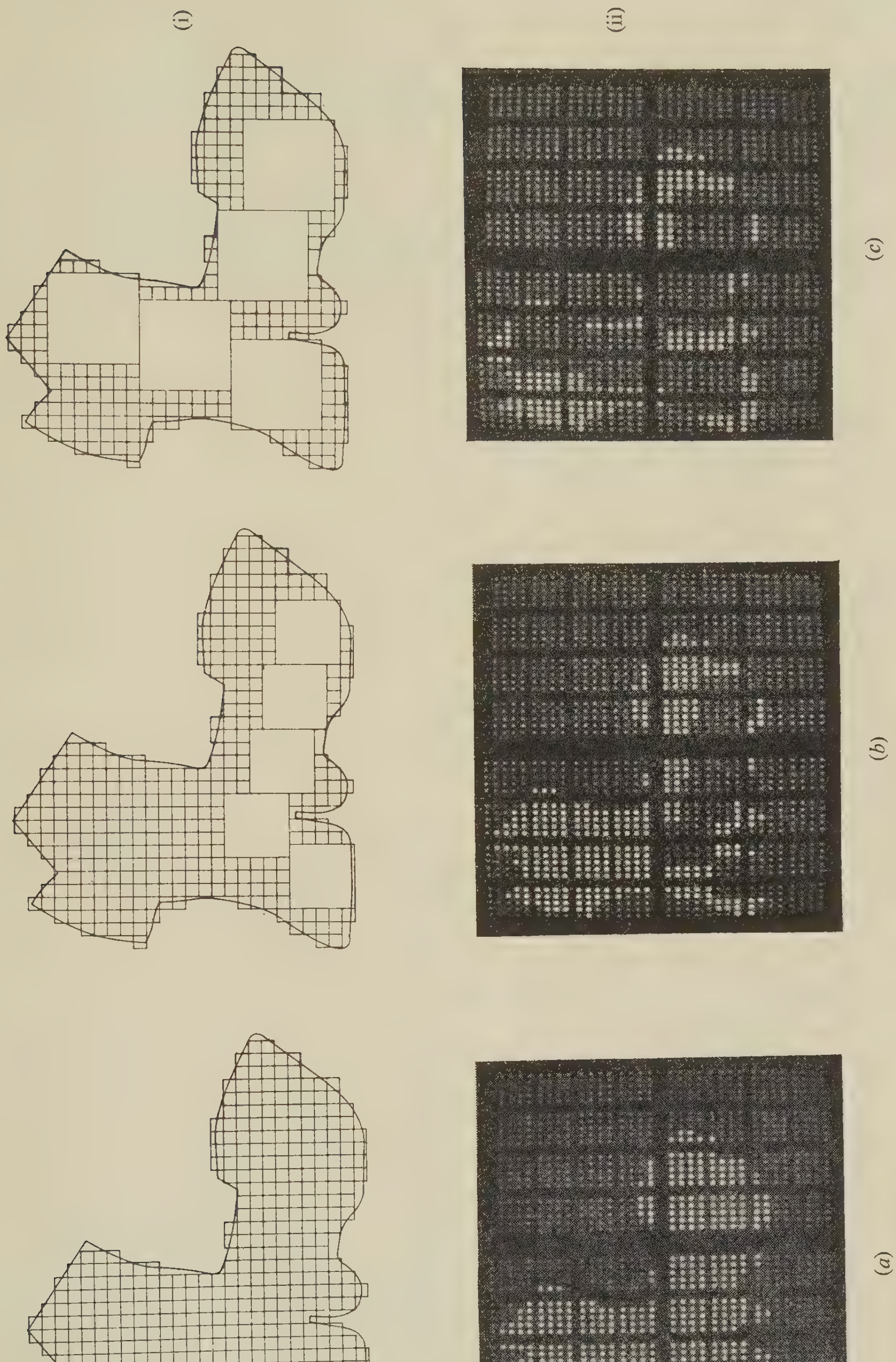

Fig. 2.—The square extraction technique.

(*a*) Complete area.
(*b*) Medium size squares extracted.
(*c*) Maximum size squares extracted.

(i) Scale drawing of estate No. 2.
□ ¼ acre unit area.

(ii) Photographs of computer console display.

new centres of gravity is repeated until there is no significant difference in the substation positions for two successive iterations.

The iterative process requires starting substation positions which are well separated, otherwise there is a danger of substation movements being restricted before optimum positions are reached.

Two methods for choosing starting positions were developed. The first originated from the idea of dividing the area into approximately equal parts, and uses a programmed scan for extracting a number of squares from the estate area. The centres of these squares are taken as starting positions. In the second method, starting points are located on the estate boundary at equal intervals.

The square-extraction technique is illustrated in Fig. 2. The complete estate, which requires five substations, is shown in Fig. 2(*a*). Starting from the bottom right-hand corner and working to the left and upwards, five small squares (3 × 3 units) are positioned. This is repeated, increasing the sides of squares by one unit [Fig. 2(*b*)] until just five squares can be contained in the area. In the case shown, the maximum size squares are 7 × 7 units [Fig. 2(*c*)]. An alternative set of starting points is obtained by repeating the square-extraction scan with the co-ordinates of the estate plan interchanged, i.e. with the estat[e] area inverted. This generally results in a different configuratio[n] of squares.

In the second method, the starting points are found by scannin[g] the border and choosing five equally spaced points on it. Thre[e] such sets of starting points are obtained by advancing the point[s] by increments around the border.

Using the square-fitting technique for the normal and inverte[d] areas and the boundary division method, five sets of startin[g] points are thus obtained. The iterative process previousl[y] described is used with all five sets in turn, and the best substatio[n] positions are chosen by retaining the result which gives th[e] minimum cost according to eqn. (2).

Two alternative minima are shown in Fig. 3, which als[o] shows the converging paths of substation positions for variou[s] starting conditions. Whilst the short paths for the square centre starting points are fortuitous for the case shown, it wa[s] generally observed that the square-fitting technique require[d] fewer reiterations and produced fewer non-optimum solution[s] than peripheral starting points.

A variant of the iterative process allows for any of the substations to be fixed and initial positions to be calculated fo[r]

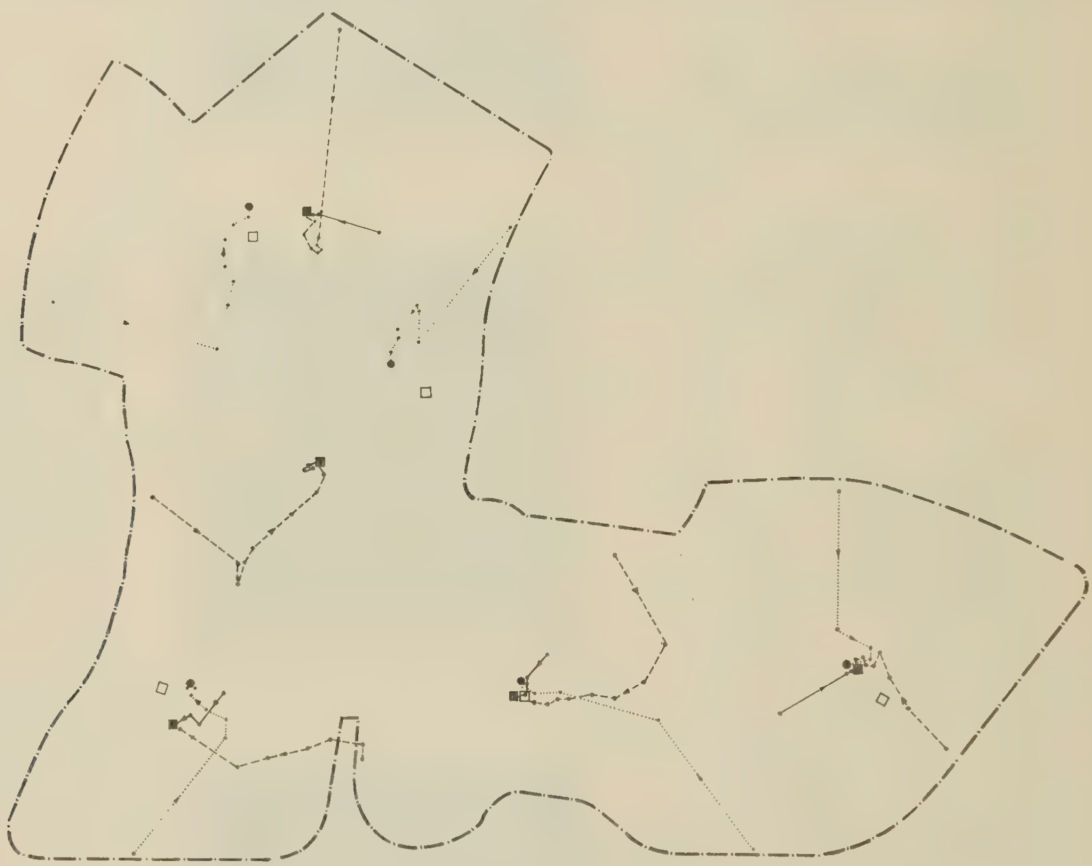

Fig. 3.—Actual and calculated substation positions for estate No. 2.

Substation positions

□ Actual.
■ Calculated (optimum).
● Calculated (non-optimum).

Converging paths for starting points at

—— Centres of squares (3 steps).*
– – – – Boundary (9 steps).*
· · · · · Boundary (8 steps).

* Optimum results.

the remainder of the substations. This procedure has been designed for three cases:

(a) Where no site near a previously calculated position can be secured from the estate developer.
(b) Where one or more of the substations for the new estate are also required for reinforcement of existing networks and therefore must be moved from the calculated position. (Alternatively, reinforcement areas can be included in the new estate area at the outset.)
(c) Where substations are fixed because large consumers agree to provide them (e.g. schools and blocks of flats).

(2.2) Feasible Substation Sites

The initial substation positions obtained by the computer will normally not be practicable, since the calculations disregard the housing and road layout. The design engineer therefore examines the initial substation positions and preselects, for each initial position, several sites within a radius of approximately 50 yd. Information concerning these sites is supplied to the computer for the cable layout calculation, and the final selection of the best sites is made automatically from the several preselected sites. If any site is fixed (Section 2.1) the location of this known site is supplied as the only one for the substation concerned.

The manual method of preselecting feasible sites was used in order to produce realistic final sites. Alternatively, the machine could have been programmed to move a substation from the initial position to the nearest road and continue the movement along roads until the minimum-cost position was obtained. But this method would still require a final manual adjustment and re-calculation to obtain accurate costs for the final sites.

(2.3) Initial Cable Layout

The flow diagram of Fig. 4 indicates the broad outlines and procedure of the cable design scheme.

(2.3.1) Selection of Feeding Routes.

The road layout is divided into road sections by using the road junctions as splitting points or 'nodes'. The information concerning load, number of consumers, length of section and a number symbol for each node is supplied to the computer for every road section (Section 7.1.2). The road network is thus presented to the machine as a list of numbers rather than by a conventional co-ordinate system, because this results in a simpler programme and speedier calculations. Adjacent sections are found automatically by inspecting the node symbols. Using the substation sites nearest the initial positions, the machine is instructed to progress from node to node in search of the shortest feeding route for every road section.

This search is in the nature of a maze problem. The machine effectively 'looks' at the plan of the estate and allocates each road section to the nearest substation.

To limit the large number of possible routes, the initial radius of search for a substation is kept small by starting with a permissible feeding length of 140 yd. (This length was found by experiment to result in the shortest computing times.) If no substation is found, the radius of search is extended in steps by increasing the permissible feeding length up to an arbitrary maximum of 700 yd. This length is, of course, unreasonable, and if ever reached it is due either to a machine fault or faulty input data. The machine then prints 'no substation' and stops.

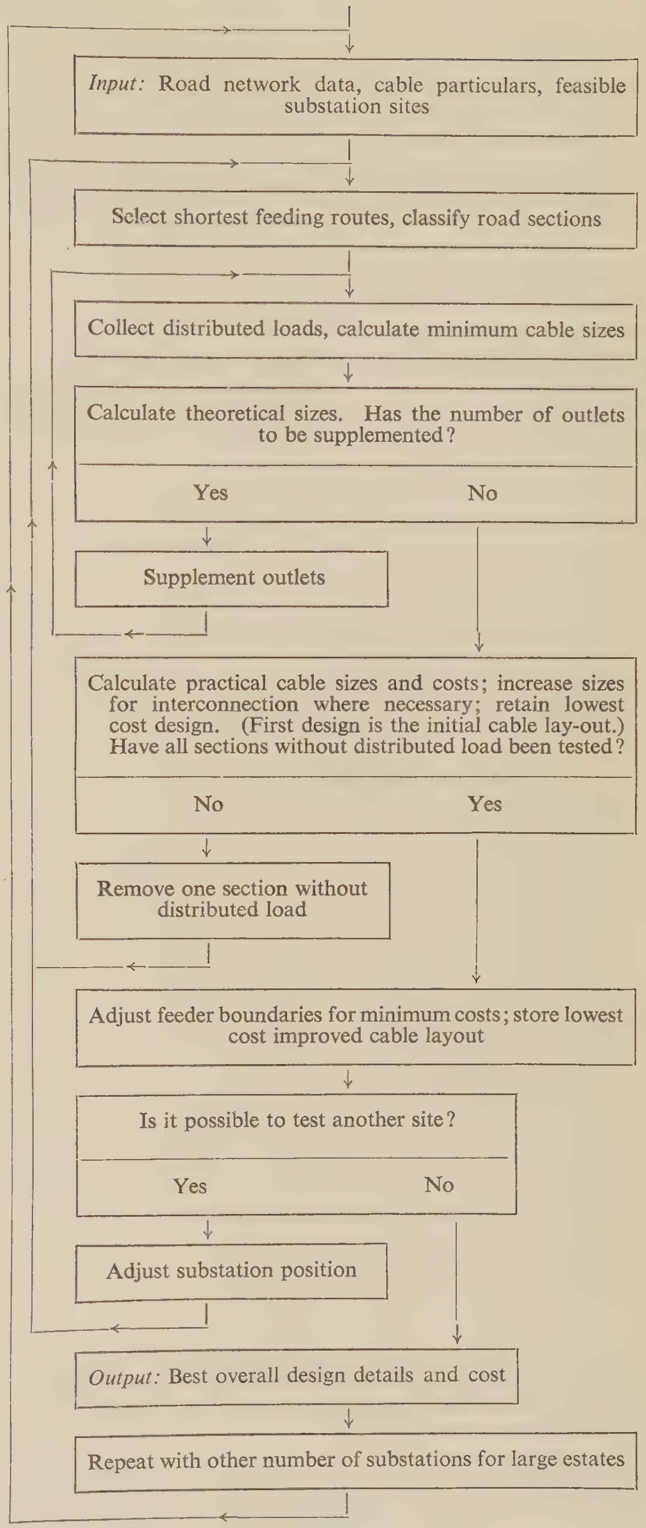

Fig. 4.—Overall flow diagram of cable design scheme.

(2.3.2) Calculation of Cable Sizes.

A radial distributor has a tree-like structure, the main stem near the substation having the largest, and the extremities having the smallest, cross-sectional areas of conductor. Opportunities to vary the size of cable exist at every node, and, as voltage drop is a limiting factor more often than current-carrying capacity, many combinations of cable sizes are possible within the distributor. Selection of the cheapest combination is therefore important.

In order to achieve rapid results the calculation is carried out in two stages:

(*a*) Theoretical sizes are calculated first to reduce the range of possibilities (Section 7.2.3).

(*b*) Practical sizes are selected next by testing for the cheapest combination of standard sizes which will not exceed a voltage drop of 6% from substation to any extremity. (The remaining 6% of the 12% total permissible range is allocated to the high-voltage system and service cables.) Only a narrow range around the theoretical size—or the minimum size for current rating if this is greater than the theoretical size—is investigated.

In practice, the optimum range was found to consist normally of only two sizes: the lower limit is either the standard size just below the theoretical size or is equal to the minimum size. The upper limit is one standard size above the lower limit. Provision to extend the upper limit is made but seldom utilized. Theoretical and minimum sizes are also used to decide when a single cable is inadequate, and, if necessary, road sections with excessive loads are supplemented by one or more parallel sections.

Interconnector requirements are considered after calculation of the practical sizes. On the basis of lowest cost, interconnectors are selected, and increased in size where necessary, to provide transfer capacity to the extent of one-third of the loading on each substation. Where cable sizes have to be increased the minimum current-rating size is replaced by the minimum interconnection size, and the practical cable sizes are recalculated, because the increase in interconnection sizes sometimes allows a reduction in sizes of other cables.

(2.4) Improved Cable Layout

The selection of routes from loads to substations is based on the principle that the shortest feed is the most economical, as theoretically it results in the lowest possible voltage drops and transmission losses.

Analysis of well-designed estates showed that this principle was generally adhered to, but two exceptions were observed:

(*a*) Where the shortest feed includes a section of road which has no distributed load of its own, and hence does not require a cable itself, it is occasionally more economical to adopt a longer feed which avoids that road section and thus saves some excavation costs.

(*b*) Because there are only a few standard sizes of cable in use, it is often impossible to utilize the maximum voltage drop completely. By inspecting the voltage drops actually achieved at the distributor ends, cost improvements can sometimes be effected by slight rearrangements of the distributor boundaries. Any variation of jointing costs caused by such an alteration must also be taken into account.

The programme attempts design improvements by first trying all possibilities under (*a*) and then (*b*).

(2.5) Best Overall Design

When the improved cable layout, which uses the sites nearest the initial substation positions, is complete, further calculations are performed to determine whether any other site gives a more economical system.

The sizes of distributor outlets are compared at every substation, and when a large size is noted in one particular direction a trial move is made to the nearest site in that direction, provided that such a site exists. If the sizes of distributor outlets are equal, the trial move is made in the direction of the most expensive distributor. After the direction for a trial move has been determined, the feeding-route selection procedure (Section 2.3.1) is employed to find alternative trial sites. Only one trial move is made at any one time and a new cable layout is designed. If the new layout is cheaper it is accepted and further trial moves to other sites for the same substation are investigated. If, on the other hand, the new layout is dearer, the old design i retained and trial moves progress to other substations. Th final cable layout therefore uses the best sites as final substatio sites and represents the optimum design within the give restrictions.

These adjustments effectively remove inaccuracies due to th approximating assumptions for the calculation of initial sub station positions. However, in the majority of cases tested, th site nearest the calculated position proved to be the best. I the few cases where other sites were accepted, no marked effec on neighbouring substations was observed.

(3) ENGINEERING AND ECONOMIC COMPARISONS BETWEEN MANUAL AND MACHINE DESIGNS

Four test designs have been calculated on the Mancheste University Mark I Computer and are compared below wit actual manual designs. Identical data, e.g. types of cable an a.d.m.d. figures, were used for both designs. Estates Nos. and 2 were manual designs for demonstrating the technique o tapered single-main systems.[2,3] Estates Nos. 3 and 4 were chose at random from available records of one Area Board.

Two separate automatic designs were prepared for each estat One design was based on the substation sites calculated by th computer (Fig. 7 and col. 4 of Table 1); the other design retaine

Table 1

Cost Comparisons between Manual and Automatic Design

Estate number	Manual design. M.V. cable and excavation *total costs*	Automatic designs. *Saving* over manual design costs		
		Minimum cost designs		Excess facilities design with actual substation sites
		Actual substation sites	Calculated substation sites	
1	£7438	£444	£484	£334
2	£11624	£440	£696	£120
3	£6939	£144	£458	£144
4	£8446	£1200	£1226	£428

the actual sites, i.e. the substations used by the designer (Fig. and col. 3 of Table 1). Two comparisons with the manua design (Fig. 5 and col. 2 of Table 1) were therefore possible:

(*a*) Observation of total improvement due to better substatio sites and better cable design.

(*b*) Observation of partial improvement due to better cable desig only.

Col. 5 of Table 1 is included because three of the four manua designs provide interconnection and control facilities in exces of requirements; the 'excess facilities costs' have been obtaine by modifying the designs using actual sites to include the sam generous interconnection and number of substation outlets a provided in the manual designs. All the savings shown i Table 1 allow for the cost of preparing automatic designs b deducting the estimated design costs (Section 3.4) from th capital savings, i.e. from the difference in cable and excavatio costs between the two methods of design.

(3.1) Substation Positions

The optimum substation sites were observed to be invariabl near the initial, calculated positions (cf. Figs. 1 and 3). Fig. also shows that the non-optimum positions obtained by one c three computations coincide closely with the actual sites. Col.

Fig. 5.—Manual design for estate No. 2.

Fig. 6.—Automatic design, using actual substation sites, for estate No. 2.

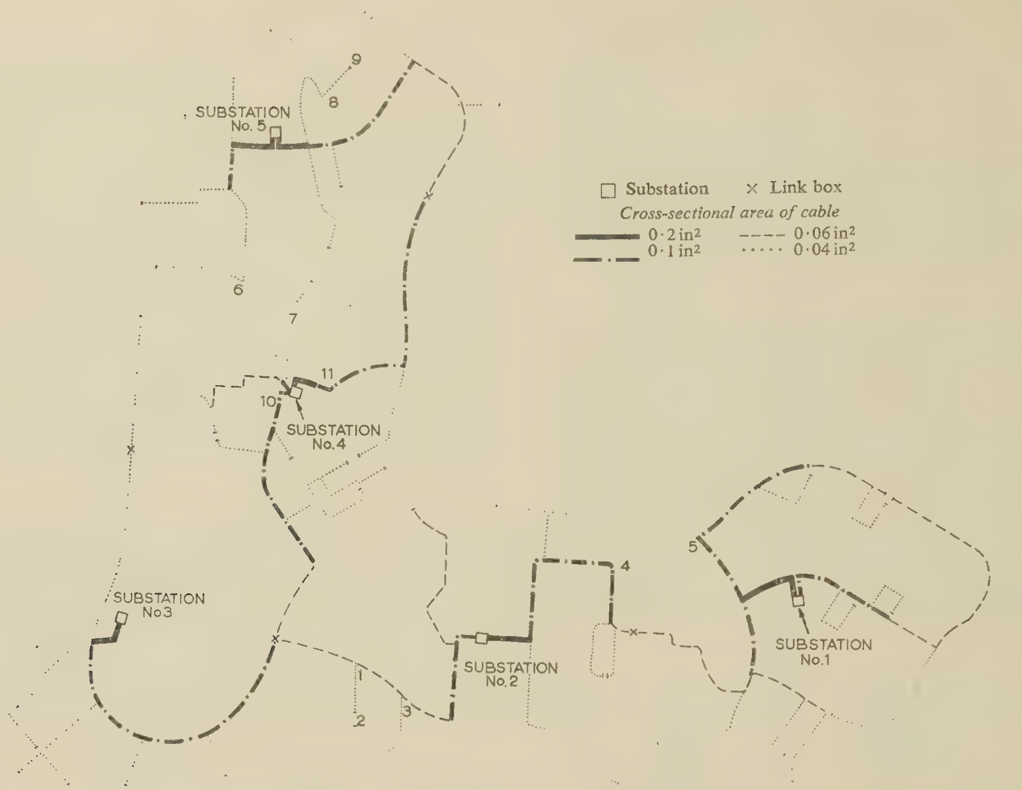

Fig. 7.—Automatic design, using calculated substation sites, for estate No. 2.

of Table 1 shows that the maximum savings were achieved in every tested design when calculated substation sites were used.

(3.2) Cable Design

It is interesting to analyse the differences between a manual cable design (Fig. 5) and an automatic cable design using the same substations (Fig. 6). There are many minor variations; for example, in road sections 1–2–3 the designer used one $0\cdot06\,in^2$ cable, but the computer showed that it was cheaper to segregate feeds at node 2 and to use two $0\cdot04\,in^2$ cables, even after allowing for increased joint costs.

More important economies are effected by the computer selecting some feeding routes which differ from the manually chosen routes. For example, the automatic design used road section 6–7, but not road section 4–5, resulting in an overall reduction of cable requirements for the feeders concerned.

Thus the computer's greater capacity for detail and its more thorough search for alternative routes achieve significant savings in cable expenditure.

Cable terminations in the automatic design extend to road nodes, whereas the manual design shows the common practice of shortening such cables to the last house block. It was not considered necessary to programme these length reductions, but they are taken into account in the present comparison.

Small variations in the feeding routes to some load blocks (e.g. road section 8–9, Figs. 1, 5 and 6) are caused by the automatic layout being confined to roads given in the data, whereas the designer occasionally decides to lay cables along paths or directly in gardens.

Until the present input methods are mechanized (Section 4) it will not be possible to retain sufficient data for the automatic programming of such deviations. However, if a cursory inspection of the results suggests that an apparent improvement is possible by introducing additional cable routes, any such modification can be readily included in the data and tested by repeating the design. Provisions for such additional cable routes are essential when feasible substations sites are chosen (e.g. section 10–11, Figs. 1, 5 and 6).

(3.3) Excess Facilities

The machine always selects the most economical number of distributor outlets from substations, whereas manual designs appear to use a criterion of maximum load, or maximum number of consumers, per outlet. This criterion results in more outlets and greater costs than automatic design for estates of low load density (e.g. estate No. 2, Figs. 5 and 6), but there is no difference in the two design methods for high-load-density estates. Various factors must be considered. With regard to the current rating it is better to install two $0\cdot1\,in^2$ cables than one $0\cdot2\,in^2$ cable, but no improvement in voltage drop is achieved, in spite of the greater cost of the two cables, since the total resistance remains constant. On the contrary, less favourable diversity due to the splitting of the load causes an increase in voltage drop. On balance, two cables are generally costlier to lay and control, and the programme therefore always selects the lowest possible number of feeder outlets.

Figs. 5 and 6 show that some cable sizes for interconnectors between substations are smaller in the automatic design. The automatic design is based on minimum engineering criteria—in this case, one-third of the substation capacity should be transferable over interconnectors—whereas design engineers tend to be more generous.

(3.4) Design Preparation Costs and Reliability

The Manchester University Mark I Computer, on which the design programme was developed, is obsolete and not sufficiently fast to make automatic design an economical proposition.

Computing costs are therefore based on faster machines such as a Mercury computer. The design programme has not yet been adapted for Mercury, but the computing times can be estimated accurately. The designs detailed in Table 1 are expected to require a range of 10 min for the simplest estate (No. 4) to 40 min for the largest estate (No. 2). Allowing for a hiring charge of £70 per hour and data preparation costs of £5 per estate, the total computing costs for estates Nos. 1–4 are thus estimated at £25, £50, £40 and £20, respectively. These costs were deducted from the capital savings to obtain the figures of savings as quoted in Table 1. The cost of manual design, on the other hand, depends not only on the size of estate, but also on the desired quality of design and the ability of the designers. It will probably vary from £10 to £20. This cost has not been taken into account in the comparison, and therefore the quoted savings are a little underestimated.

Many safeguards are 'built into' the programme to stop the machine when the programme departs from its designed sequence owing to a computer breakdown. These safeguards act very rapidly, and therefore final results are almost always error-free. In contrast, the comparison revealed some errors in the manual designs.

(4) CONCLUSIONS AND POSSIBLE DEVELOPMENTS

The automatic method of design has shown that the experienced engineer often achieves manual designs which are similar in many respects to automatic layouts. However, detailed comparisons reveal the computer designs to be more economical. Maximum savings ranged from 6 to 15% of the capital expenditure when automatic designs, using calculated sites, were compared with manual designs. Use of the actual, previously chosen sites in the automatic cable design programme still produced some savings. Even when adjustments were made to the automatic design to provide the same excess facilities as the manual design—which were not necessary to comply with accepted standards of supply—the cable layout was still better and cheaper than the manual design. Furthermore, the automatic design saves skilled labour and achieves uniform, high standards of design, irrespective of any cost reductions achieved.

The automatic method can be used to study the economics of variations in basic design criteria. For instance, the number of cables can be varied in a study of the optimum cable standards. As an example, the design of estate No. 1, which initially used six cable sizes (0·0225, 0·04, 0·06, 0·1, 0·15 and 0·2 in^2) was repeated with five sizes, omitting the 0·0225 in^2 cable. This resulted in an increased cost of £190. Another redesign with four sizes (omitting the 0·15 in^2 cable as well) increased the cost further by £320. No definite conclusions can, of course, be drawn without subjecting a large number of estates to such tests and also investigating overhead costs, but the results suggest that a 5-cable standard might be the optimum.

Preliminary investigations show that an automatic scanning system[4] could be used to read the data directly from a map. Such a system would considerably facilitate the preparation of data; furthermore, it would enable the design to be made completely automatic, since it would be possible to programme both the preselection of feasible substation sites and the creation of cable routes additional to the road network when such routes were necessary.

Alternatively, the preselection of sites would be unnecessary with a new construction technique using underground transformers. These could be placed anywhere on the road network, and would result in better layouts and further substantial savings. In present practice, a plot of land must be secured for every transformation point and it is not certain that the best sites chosen by the computer will be secured from the estate developer. Therefore, the savings quoted may not, in fact, be achieved. On the other hand, automatic design should prove of considerable help during negotiations, since the results could be used to substantiate the case for good sites by quoting the exact extra cost for less favourable sites.

Consideration of the cost of transmission losses could be readily incorporated in the programme and the high-voltage distribution design could also be included. However, in the four tested designs, neither the cost of l.v. losses nor the h.v. cable costs proved to be significantly different for the two design methods.

Ideas for route selection between load and substation might find other applications, and it may be possible that the methods for siting substations will find uses in other distribution problems which involve a division of irregular areas. The iterative process described could be used to locate substation positions for reinforcing existing networks.

(5) ACKNOWLEDGMENTS

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(7) APPENDICES

(7.1) Details of Data

(7.1.1) Substation Calculations.

For the initial substation calculations, one line of data represents the co-ordinates and loading of one unit area ($\frac{1}{4}$ in^2 on 1/2500 scale, or $\frac{1}{4}$ acre) as follows:

x co-ord.	*y* co-ord.	Load, kW
XX	XX	XX

X stands for any figure 0 to 9 inclusive. For example, a line of data given as 000309 refers to unit area $x = 00$, $y = 03$ containing a 9 kW load.

Information concerning the feasible substation sites takes the form of one 6-figure number, specifying the node, and a 2-figure number, specifying the substation number, for each site.

(7.1.2) **Cable Design.**

The following information is given for every road section:

Node symbol at one end	Node symbol at other end	Length	Load kW	No. of consumers
XXXXXX	XXXXXX	XXX	XXX	XXX

Nodes are described by consecutive simple numbers 1, 2, 3, etc. For a possible future automatic intake of information six figures are allowed, however, to facilitate a correct positioning in space by means of two 3-figure x and y co-ordinates. The length is in units of $\frac{1}{32}$ in on a 1 : 2 500 scale map (equivalent to 2·18 yd). The load in kilowatts and the number of consumers occupy the last two groups of three figures.

(7.1.3) **Constants.**

Costing information and cable particulars are contained in a 'constants' tape. On-costs are added and all constants are printed out as a check prior to the start of the design programme. An example of the computer check print is shown in Table 2.

Table 2

Explanations	*Computer Print*		
Copper (C) or aluminium (A)	C		
Number of standard cables used	4		
Size (in²), price with on-cost (£ per 1 000 yd)	0·04	785	106
and capacity (kW)	0·06	966	135
	0·10	1 444	181
	0·20	2 363	276
Substation cost, £	500		
Excavation price with on-cost, £ per 1 000 yd	506·25		
Straight joint price, £	12		
Material on-cost, %	17·15		
Labour on-cost, %	26·50		

(7.1.4) **Restrictions in Data and Constants.**

The length of a road section is virtually unlimited, but as there is no automatic division of very long sections it is preferable to separate lengths in excess of 64 units (139 yd). For accuracy, a road section must be divided if the density of its distributed loading changes significantly. A maximum number of 248 road sections can be stored, and no more than four road sections may meet at one point. The total load on a substation must not exceed 1 MW, and no more than 12 substations can be accommodated. It is possible to use up to six standard cable sizes of any size up to 1 in².

Most of these restrictions are necessary to avoid exceeding the computer storage capacity. They have been so chosen that there is little likelihood of interference with the most extensive schemes.

(7.2) Design Scheme

A detailed description of the programme will be found in Reference 5, but some mathematical and logical details are given below.

(7.2.1) **Classification of Road Sections.**

Road sections are defined, and marked by special digits:

End sections.—A road section ending in a cul-de-sac.

Split section.—Where the two ends or nodes belong to different distributors, the correct dividing point is determined by making the distances from substation to dividing point equal for both distributors.

Pseudo end section.—To avoid short pieces of cables near road junctions the distance from section end to dividing point is tested, and if it is less than 12 units (25 yd) the dividing point is moved to the end.

Through sections.—A road section which serves to connect any of the above sections to a substation.

Substation sections.—A section with a substation at one end.

No-cable sections.—Any split section or end section which contains no distributed load is now marked as 'redundant', since no cable will be required for it. Through sections having neither distributed load nor through load are treated similarly.

Through sections without distributed load, but carrying through loads, are specially noted for future possible transfers of their through load to other sections.

(7.2.2) **Load Calculations.**

Effects of diversity and unbalance are treated according to Reference 2, which specifies two multiplying factors for unbalance and diversity; these are combined in the design programme. The two factors are not applicable to large consumers, who are often restricted by agreement to a specified demand and normally maintain a reasonable phase balance. A load/(number of consumers) ratio test is included for this reason, and if the ratio exceeds 10 the combined factor is taken as unity.

A derating factor is applied to the total section load, and the smallest cable which can carry this adjusted load is selected as the minimum size. A special mark is inserted if no standard size has a sufficiently large rating, and these specially marked sections are later reinforced by one or more newly created road sections.

(7.2.3) **Theoretical Sizes.**

Karapetoff[6] derived an 'equivalent feeder' method to calculate the most economical cross-sectional areas of conductors, based on voltage-drop considerations only, for radial feeders with point loads at their extremities. This method was adapted for systems with distributed loads by introducing an imaginary feeder of zero length at the extremity of each feeder carrying a distributed load.

Fig. 8 shows such a feeder system. Two point loads P and Q, requiring currents I_1 and I_2, respectively, are supplied from a

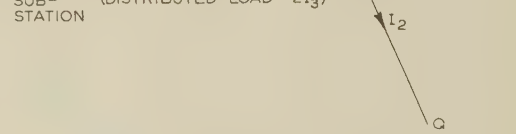

Fig. 8.—Theoretical radial feeder.

substation by feeders 1 and 2 of length l_1 and l_2, respectively, and the common feeder O of length l_0.

Let I_3 be the terminal load on an imaginary feeder 3 (feeder O–S in Fig. 8), replacing a distributed load of twice the value of I_3 in feeder O, and let l_3 be the length of the imaginary feeder, equal to zero.

The amount of conductor metal will be a minimum if the total permissible voltage drop at the extremities (points P, Q

and S) is fully utilized. This requires the choice of the optimum voltage drop at the junction of the four feeders (point O).

Let a_0, a_1, a_2, a_3, be the best theoretical sizes of cables for feeders 0, 1, 2 and 3 respectively.

As in the case of eqn. (2), it is assumed that the cost is proportional to the volume v of the conductors, which is given by

$$v = a_0 l_0 + a_1 l_1 + a_2 l_2 + a_3 l_3 \quad . \quad . \quad . \quad . \quad (3)$$

and this should be a minimum.

Considering a single wire of a 3-phase system and choosing the voltage drop at P, Q and S to be the maximum permissible voltage drop V, let the unknown voltage drop at the point O be V_x.

Then
$$V - V_x = \frac{\rho l_1 I_1}{a_1} = \frac{\rho l_2 I_2}{a_2} = \frac{\rho l_3 I_3}{a_3} \quad . \quad . \quad . \quad (4)$$

$$V_x = \frac{\rho l_0 (I_1 + I_2 + I_3)}{a_0} = \frac{\rho l_0 I_0}{a_0} \quad . \quad . \quad . \quad (5)$$

Substituting eqns. (4) and (5) into eqn. (3), differentiating and equating to zero for minimum, we have

$$\frac{l_0(I_1 + I_2 + I_3)^{1/2}}{V_x} = \frac{(l_1^2 I_1 + l_2^2 I_2 + l_3^2 I_3)^{1/2}}{V - V_x}$$
$$= \frac{(l_1^2 I_1 + l_2^2 I_2)^{1/2}}{V - V_x} \quad . \quad . \quad . \quad . \quad (6)$$

since $l_3 = 0$

The two feeders 1 and 2 may be replaced by an 'equivalent feeder' O-R of length l_0', carrying current $(I_1 + I_2 + I_3)$ and giving the same voltage drop $(V - V_x)$ as each individual feeder,

where
$$l_1^2 I_1 + l_2^2 I_2 = l_0'^2 (I_1 + I_2 + I_3) \quad . \quad . \quad . \quad . \quad (7)$$

Then
$$l_0' = \left(\frac{l_1^2 I_1 + l_2^2 I_2}{I_1 + I_2 + I_3}\right)^{1/2} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Length l_0' is added to l_0, resulting in a fictitious feeder which may be treated exactly as a normal feeder. The theoretical size a_0 can now be calculated and the voltage drop at point O found by proportion. Finally, theoretical sizes a_1 and a_2 using the remainder of the allowable voltage drop can be obtained.

(7.3) Computer Output Details

(7.3.1) Initial Substation Calculations.

The printed output, with explanatory remarks, for test design No. 2 is shown in Table 3.

Substation positions are given by x and y co-ordinates, and loads on each substation, as well as total load, are recorded.

Table 3

RESULTS OF INITIAL SUBSTATION CALCULATIONS

Explanations:	*Computer print:*
Area as number of ¼ acre squares	451
Total load, kW	1 734
Number of substations calculated	5
Number of squares fitted ..	5
Substation numbers, co-ordinates and loads	No. 1: $x = 9{\cdot}75$; $y = 7{\cdot}51$; 449 kW No. 2: $x = 10{\cdot}00$; $y = 15{\cdot}01$; 300 kW No. 3: $x = 16{\cdot}00$; $y = 22{\cdot}26$; 334 kW No. 4: $x = 5{\cdot}50$; $y = 23{\cdot}00$; 325 kW No. 5: $x = 26{\cdot}25$; $y = 27{\cdot}52$; 326 kW
Number of iterations ..	3
Σ load $\times (x^2 + y^2)$ ..	1414·656

(7.3.2) Best Overall Design.

Table 4 shows parts of the printed output of cable layout and substation positions for test design No. 2, and also gives an explanation of the abbreviated headings.

Table 4

Explanations

Node one end	Node other end	Length in $\frac{1}{32}$ in units	Load	Sub-station number	Cable size in^2	Cumulative voltage drop

Computer print

x_1	y_1	x_2	y_2	l	kW	SS	A	V_D
0	1	0	2	28	12	2	0·04	5·13
0	2	0	3	40	24	2	0·04	5·38
0	4	0	5	46				
0	6	0	7	24	28	5	0·04	5·39
0	8	0	9	20	8	5	0·04	4·64
0	46	0	65	14	11	4	0·04	6·02
				26	21	2	0·10	6·06
0	120				332	1		
0	54				353	2		

DR No. 1: £1 147.
DR No. 2: £1 215.
. . .
DR No. 10: £1 433.
DR No. 11: £930.
Total: £11 644.

Lines with no output apart from nodes and length represent road sections where no cable is required (section 04–05, also shown in Fig. 6).

'Split sections' (i.e. road sections which contain a splitting point between two feeders) are given by a pair of lines, the second of which has no co-ordinates (section 046–065).

Final substation sites are recorded in lines which specify one node only, together with the load on the substation and its reference number.

Costs for each distributor DR and total costs are recorded at the end of the output. This total is the computed cost without the adjustments detailed in Section 3. To arrive at the figures of savings (Table 1) the computing costs must therefore be added. The estimated excess costs for distributors extending to road nodes, instead of terminating at the last house, must be subtracted from the total cost.

[The discussion on the above paper will be found on page 314.]

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THE LOGICAL DESIGN OF ELECTRICAL NETWORKS USING LINEAR PROGRAMMING METHODS

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(*The paper was first received* 16*th March, and in revised form* 10*th July*, 1959. *It was published in December*, 1959, *and was read before the* SUPPLY SECTION 10*th February*, 1960.)

SUMMARY

The results of an investigation into the mathematical design, as distinct from analysis, of electrical power-system networks are described.

Starting with geographical positions of the substations which it is required to interconnect, it is shown that a set of equations can be obtained which are solvable by linear-programming techniques to obtain a minimum-cost network design. Any security of supply conditions considered necessary can be incorporated into the design equations.

Solution of the resulting linear programmes requires the use of a digital computer; the necessary computer size, and amount of computation, increases rapidly with increase in the number of substations to be interconnected. With this in mind, suggestions for increasing the size of problem solvable on a given computer are made.

Three designs have been completed using the method proposed. These and the results of network-analyser studies on two of them are summarized.

The equations for two other possible network-design criteria, minimum circuit length and minimum apparent-power by distance product, are also given and briefly commented upon.

LIST OF PRINCIPAL SYMBOLS

$S_1 \ldots S_m$ = Substations at which power is supplied to the network under consideration, referred to as supply substations.
S_s = General supply substation.
$S_n \ldots S_r$ = Substations at which power is taken from the network under consideration, referred to as load substations.
S_l = General load substation.
S_i, S_j = Any of a group of substations irrespective of whether power is supplied to or taken from the network under consideration.
p_{ij} = Permissible circuit path between substations S_i and S_j.
l_{ij} = Distance between substations S_i and S_j.
c_{ij} = Cost of one circuit between substations S_i and S_j along path p_{ij}.
S = Maximum rating of circuits on proposed network.
S_j = Apparent power transfer at substation S_j.
f = Network cost function to be minimized.
g = Network apparent power by distance function to be minimized.
m = Total number of supply substations.
n = Total number of load substations.
s = Number of substations in a group.
h = Number of circuits required into a group of s substations.
S_L = Total apparent load in a group of substations.

Mr. Knight was formerly with the Merseyside and North Wales Electricity Board, and is now with the Central Electricity Generating Board.

(1) INTRODUCTION

Many papers and books have been written over the past sixty years on the analysis and prediction of the performance of electrical power-system networks. D.C. and a.c. network analysers have been used to supplement hand computation in steady-state network analysis since about 1925, and some analysers have been built to investigate transient phenomena. Since 1952, digital computers have been increasingly employed to obtain numerical solutions to steady-state and transient network equations.

However, all this work has been concentrated on analysing the performance of networks which exist, even if only on paper. Very little has been done on the mathematical design of networks. To illustrate the point, a network may be proposed which has a circuit between two substations A and B. Existing methods enable one to predict what the voltage and power-flow conditions on this circuit are likely to be and what, for instance, its cross-section should be. They do not give any assistance in saying whether, in fact, a circuit should be provided between A and B rather than, say, A and C, except by analysing and costing the two networks independently. Only a limited number of designs are usually studied in any detail, and for complicated networks, which from the point of view put forward here means those required to connect a number of substations, one can never be sure that the end-product of this work is, in fact, the cheapest scheme that would provide the service required.

The paper suggests a method of obtaining mathematically a minimum-cost outline design of a network to supply a number of load points from a number of supply points. The method is based on the use of the technique of linear programming, developed by economists and mathematicians during and since the last war. This technique has been used to solve problems in a wide range of industries, such as the blending of fuels, the mixing of animal feeding stuffs and the transport of coal between pits and power stations.[1, 2] The problem involves the consideration of a number of variables whose relationship with each other is defined by a set of linear equations (or constraints), the number of variables exceeding the number of equations, and subject to the general condition that the variables should be non-negative. There may be a large number of solutions satisfying the constraints, and the problem is to find which of these solutions has some preferred characteristic, say minimum cost. Normal algebraic methods cannot be used to solve such a problem, and linear programming enables the preferred or optimum solution first to be obtained in a systematic manner and secondly to be identified when it is obtained. The amount of arithmetic work is frequently such that computers must be used to obtain a solution.

(2) RELATIONSHIP OF LINEAR PROGRAMMING AND NETWORK DESIGN

(2.1) Requirements of a Power-Supply Network

A power-supply network is designed so that it will transmit given amounts of electrical power subject to the following conditions:

(*a*) The cost of the network to construct and operate should be as small as possible.

(*b*) The continuity of supply afforded by the network should not be less than the minimum acceptable for loads of the size and type connected to the network.

(*c*) In the case of a generating station, the connections provided should give adequate capacity out of the station under those circuit outage and load conditions assessed as technically and economically justified during the design study.

(*d*) The necessary operational and control facilities required to obtain satisfactory performance from the network should be consistent with the facilities normally available for a network supplying loads of the size and type concerned.

(*e*) Extension of the network should be possible.

(*f*) The technical design of the network should be adequate, i.e. there should be no risk of harm to plant or personnel under normal or fault conditions.

Conditions (*d*) and, to a lesser extent, (*e*), are expressions of opinion and not readily amenable to analysis. Given a network design on paper it can be studied, if necessary with the aid of any analyser or digital computer available, to ensure that it complies with condition (*f*).

It will be shown that linear programming (l.p.) techniques can be used in the attainment of conditions (*a*)–(*c*).

(2.2) Difficulties in Network Design

Even in Britain at the present time, one is sometimes faced with the problem of interconnecting a number of substations by a network which will comply with the conditions given in Section 2.1, there being no existing network. Such cases may arise in:

(*a*) The provision of distribution networks (medium and high voltage) to supply new housing estates.

(*b*) The reinforcement of existing distribution networks by superimposed subtransmission networks.

(*c*) The reinforcement of existing subtransmission networks by superimposed transmission networks.

In each case, one is presented with a geographical disposition of substations which require connecting together at minimum cost subject to given conditions for security of supply, etc. As the number of substations increases, the ways in which these connections can be made in a technically satisfactory manner will become large, and the art of system design lies in choosing the scheme which is both technically and economically the best, or at least makes a reasonable compromise between the two requirements.

In order to tackle the network design problem logically rather than intuitively, some means is required of ensuring that all technically satisfactory schemes are considered, and all but the optimum one (probably from the point of view of cost) rejected.

Linear programming enables this to be done, and subject to any limitations written into the design equations by the design engineer, one can say that the l.p. solution will, in fact, be the cheapest design.

(2.3) Linear Programming

Consider a set of variables $x_1, x_2, \ldots x_u$, related by v linear equations with $u > v$. An infinite number of sets of solutions exist for the x_j. Linear programming enables that set of non-negative values of the x_j to be selected which will minimize (or maximize) a linear function of the x_j.

Expressed in equation form, if a variable x_j is related by the following equations:

$$\left.\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1j}x_j + \ldots + a_{1u}x_u = b_1 \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{ij}x_j + \ldots + a_{iu}x_u = b_i \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ a_{v1}x_1 + a_{v2}x_2 + \ldots + a_{vj}x_j + \ldots + a_{vu}x_u = b_v \end{array}\right\} \quad . \quad (1)$$

$$x_j \geqslant 0$$

a minimum or maximum of F can be obtained where

$$F = c_1x_1 + c_2x_2 + \ldots + c_jx_j + \ldots c_ux_u \quad . \quad . \quad (2)$$

Frequently the constraints [eqn. (1)] are in the form of inequalities rather than equations. They may be lower-bounded inequalities

$$\sum a_{ij}x_j \geqslant b_i \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1a)$$

or upper-bounded inequalities

$$\sum_j a_{ij}x_j \leqslant b_i \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1b)$$

In either case they can be converted into equations by the addition of slack variables, x_q, as follows:

$$\sum_j a_{ij}x_j - x_q = b_i \quad . \quad . \quad . \quad . \quad . \quad (1c)$$

or

$$\sum_j a_{ij}x_j + x_q = b_i \quad . \quad . \quad . \quad . \quad . \quad (1d)$$

The function to be optimized is also modified to

$$F = \sum_j c_jx_j + \sum_q 0x_q \quad . \quad . \quad . \quad . \quad . \quad (2a)$$

As the coefficients of the x_q are zero in F, these x_q can take any value [thereby permitting the 'greater than' or 'less than' signs in eqns. (1*a*) and (1*b*) to be effective] without directly affecting the value of F.

One of the best-known ways of carrying out this optimization process—the simplex method—is due to Dantzig[3] and is described with algebraic proof by Vajda.[4]

Without going into any detail, in this method the constraint equations are written down in matrix form, known as a simplex tableau. This tableau or matrix is transformed by simple rules into a second tableau which gives a slightly nearer optimum solution for F. The process is repeated until a final tableau is obtained which gives the optimum solution. The reaching of this optimum solution is known from the appearance of the tableau. A considerable number of iterations (successive tableaux) may be necessary.

(3) FORMULATION OF NETWORK-DESIGN EQUATIONS

Three criteria have been used in the formulation of design equations for subsequent solution by l.p. methods. These are:

(*a*) A criterion which gives a minimum-cost design.

(*b*) A criterion which gives a network design with a minimum product of apparent power and distance.

(*c*) A criterion which gives a design with minimum circuit length.

The first of these is considered in some detail in Sections 4–8. The others are mentioned briefly in Section 9.

(4) NETWORK DESIGN BY THE MINIMUM-COST CRITERION

(4.1) Outline of Method

Before attempting to formulate any design conditions mathematically, it is necessary to consider what supply conditions the proposed network has to satisfy. These will depend primarily on the network voltage. Thus, on medium-voltage networks it is not always economical to provide a full duplicate supply, but particular attention must be paid to the voltage regulation. On the higher-voltage networks a failure of the network will involve many consumers, and duplication of supply is more important. Network voltage regulation is less important, although at the highest transmission voltages, where long transmission distances are encountered, it and the associated problem of supplying reactive power to the network at the correct points

must be considered. In general, the method described is considered to be particularly applicable to the design of high-voltage distribution networks, subtransmission networks and the lower-voltage transmission networks.

At these voltages, the maintenance of supply under any likely combination of conditions is probably the most important consideration. Other factors to be considered may be the avoidance of excessive expenditure on switchgear and of too many circuits along any one route. These points will be considered in detail in the following Sections.

As a starting-point it is assumed that unless the terrain (natural or man-made) prevents it, a path p_{ij} for one or more circuits exists between every pair of substations S_i and S_j. The aim of the subsequent work is to assign a value 0, 1, 2, . . . to each p_{ij} indicating that the optimum design requires 0, 1, 2, . . . circuits between substations S_i and S_j.

In order to do this, inequalities specifying security conditions and any other design conditions considered necessary, such as limitation of the number of circuits into substations or along any given routes, are written down in terms of the possible paths. These linear inequalities are then used as constraint inequalities subject to which a cost function of the network, again in terms of the possible paths, is minimized. Generally the values obtained for the p_{ij} will be non-integer, and it is necessary to employ some method which will produce an integer-valued solution from the non-integer one. An algorithm devised by Gomory[5] has been used for this purpose.

(4.2) Security of Supply

When writing the security of supply inequalities it is necessary to bear in mind that, at this stage, one has no knowledge of what the final network connections will be. It is therefore necessary to specify minimum connections to every possible group of substations to ensure that all groups in the final design will have adequate circuit capacity connected to them.

Many supply authorities have standardized transformer sizes and overhead-line and cable ratings. On some types of network, particularly distribution and subtransmission networks, it is possible to say that 2 circuits will supply up to, say, 3 substations, 4 substations will require 3 circuits, and so on. Alternatively, if substation loads differ appreciably, and particularly if the design is to incorporate connections to generating stations, it is necessary to estimate the circuit capacity required into every possible group of substations from the actual load and/or generating plant at each substation.

Assuming for the moment that the substations on the network to be designed are approximately equally loaded, the security of supply conditions can be specified as

(a) Each load substation must have at least h_1 circuits connected into it.

(b) Each possible group of 2 load substations must have at least h_2 circuits connected into it.

(c) Each possible group of 3 load substations must have at least h_3 circuits connected into it.

and so on for every possible group of load substations of all sizes up to and including all the load substations.

This set of conditions will lead to the following inequalities:

$$\sum_{i=1\neq l}^{r} p_{il} \geqslant h_1 \text{ for load substation } S_l; \quad n \leqslant l \leqslant r \qquad (3)$$

$$\sum_{i=1\neq l_1,l_2}^{r} p_{il_1} + \sum_{i=1\neq l_1,l_2}^{r} p_{il_2} \geqslant h_2 \text{ for load substations } S_{l1}, S_{l2}; \quad n \leqslant l_1, l_2 \leqslant r \qquad (4)$$

$$\sum_{i=1\neq l_1,l_2,l_3}^{r} p_{il_1} + \sum_{i=1\neq l_1,l_2,l_3}^{r} p_{il_2} + \sum_{i=1\neq l_1,l_2,l_3}^{r} p_{il_3} \geqslant h_3 \text{ for load substations } S_{l1}, S_{l2}, S_{l3}; \quad n \leqslant l_1, l_2, l_3 \leqslant r \qquad (5)$$

If the proposed network is also to interconnect generating stations, it is necessary to add a set of equations to specify that each generating station, every group of generating stations and every group of generating stations and load substations has sufficient circuit capacity connected to it to ensure that generation is not restricted by lack of circuit capacity.

When the network is to be supplied only from one supply substation, it is not necessary to specify any circuit connections at this substation, as the final security of supply equation (for all the load substations) will dictate what total circuit capacity is required into all the load substations and therefore out of the supply substation. If more than one supply substation is to be connected to the network, it may be necessary to specify at least a certain number of circuits out of each of the supply substations.

(4.3) Limitation of Switchgear

Various methods have been adopted by supply engineers to reduce capital investment in switchgear, which may well account for 40–50% of the cost of a substation. These methods usually take the form of controlling more than one piece of equipment from one circuit-breaker, and, more important from the aspect of network design now being considered, almost always limit the number of circuits which can be controlled at one substation. This limit may be 2, 3 or 4 circuits (excluding local transformers). An analysis of the number of circuits controlled at load substations in one large distribution area has, in fact, shown that about 75% of all load substations have no more than 4 circuits connected into them.

If, therefore, it is desired to limit the maximum number of circuits controlled at any load substation to, say, k, a set of inequalities can be written down as follows:

$$\sum_{i=1\neq l}^{r} p_{il} \leqslant k \text{ for load substation } S_l \qquad (6)$$

This type of inequality could also be used to ensure that excessive fault levels would not occur on the proposed network when the fault infeed over circuits is known to be very approximately constant, as is often the case with meshed network development.

Supply substations must be focal points of a network, and hence any preconceived limitation of the number of circuits connected into these is not justified except perhaps for fault-level control.

(4.4) Limitation of Inter-Substation Circuits

To ensure that all circuits shall provide as much opportunity as possible for connection into future substations, many supply engineers consider that the number of circuits along any route should be restricted. This stipulation will lead to a set of inequalities of the form

$$p_{ij} \leqslant w \qquad (7)$$

where w is the number of circuits along any one route which it is not wished to exceed.

If substations S_i, S_j and S_k are practically in line, inequalities of the form

$$p_{ij} + p_{ik} \leqslant w \qquad (7a)$$

$$p_{ik} + p_{jk} \leqslant w \qquad (7b)$$

will again ensure that a given number of circuits along paths between these substations is not exceeded.

(4.5) Network Cost Function

The network cost function which has to be minimized will be

$$f = \sum_{i=1 \neq j}^{r} \sum_{j=1 \neq i}^{r} c_{ij} p_{ij} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

This equation implies that the cost of providing circuits between two points is proportional to the number of circuits. This is true for single-circuit overhead lines, is practically true for underground cables, but will only be very approximately true for multi-circuit overhead lines.

The cost, c_{ij}, of a circuit between substations S_i and S_j should include the cost of the controlling switchgear plus a proportion of the establishment and civil engineering costs for substations S_i and S_j. It is necessary, therefore, to assume the number of circuits to be connected into a substation in the final design in order to apportion the establishment and part of the civil costs. If this assumption is incorrect, it is unlikely to affect the final design, as the error in the cost proportion will be small in relation to the total circuit cost.

(5) SOLUTION OF THE DESIGN EQUATIONS

(5.1) Linear Programming by Digital Computer

Practical l.p. problems are essentially concerned with large numbers of variables and equations, since in small applications the intuitive approach backed by experience is often adequate. Hence the practical application of linear programming frequently requires access to a digital computer.

A number of digital computers have been programmed to solve l.p. problems, mostly using Dantzig's simplex procedure or some modification of it.[6] The limiting feature in the size of problem which can be solved is the computer storage, as the l.p. solution requires the storage and manipulation of a matrix slightly greater than the number of equations in one axis and the number of variables in the other axis.

If a_1 is the number of equations and upper-bounded inequalities and b the number of variables, a computer solution can be obtained by the reduced simplex method if

$$(a_1 + \text{small integer})\,(b + \text{small integer}) < \text{computer storage} \quad . \quad . \quad . \quad (9a)$$

with, frequently, individual maximum values specified for a_1 and b. Lower-bounded inequalities must first be converted to equations by adding extra variables (Section 2.3) which must be included in the total for b.

When the constraints are in the form of lower-bounded inequalities, the dual simplex method has advantages. Using this, the problem solvable will be defined by

$$(a_2 + \text{small integer})\,(b + \text{small integer}) < \text{computer storage} \quad . \quad . \quad . \quad . \quad (9b)$$

where a_2 is the number of lower-bounded inequalities and b the number of variables.

A survey of computer designs published recently[7] showed that the largest British computers now available have internal storage capacities of up to 60 000 words. Storage capacity can be increased many times by the addition of external facilities such as magnetic tape, but there seems to be little information available on what computation times will be required to solve l.p. problems using such storage facilities.

(5.2) Network-Design Matrix Size

With m supply and n load substations to be connected to the network, there will be ${}^{m+n}C_2$ possible paths between the substations, or the number of paths will be

$$\tfrac{1}{2}(m+n)(m+n-1) \quad . \quad . \quad . \quad . \quad (10)$$

If all the design conditions proposed are to be included, there will be n upper-bounded inequalities specifying maximum numbers of circuits at each load substation, and ${}^{m+n}C_2$ upper-bounded inequalities specifying maximum numbers of circuits along any path. There will be ${}^{n}C_1$ lower-bounded inequalities specifying security conditions to each load substation, ${}^{n}C_2$ inequalities specifying security conditions to all possible groups of two load substations, and so on. Hence the total number of lower-bounded inequalities required to specify security conditions is

$${}^{n}C_1 + {}^{n}C_2 + {}^{n}C_3 + \ldots + {}^{n}C_{n-1} + {}^{n}C_n = 2^n - 1 \quad . \quad (11)$$

The total number of constraint inequalities will therefore be

$$(n + {}^{m+n}C_2)_{u.b.} + (2^n - 1)_{l.b.}$$

in terms of ${}^{m+n}C_2$ variables.

A network design to supply 8 load substations from 2 supply substations would require the setting down of 308 inequalities in terms of 45 possible paths.

Experience in the application of this method of network design so far has indicated that the majority of load substations in the final design do, in fact, have the minimum number of circuits connected and that the circuits per path will rarely exceed two. It is considered from this that inequalities limiting circuits into substations and along paths are, in practice, unnecessary. It can be argued that the fewer the conditions imposed on the design initially the better.

Even so, by far the greatest number of constraint inequalities result from the specification of minimum numbers of circuits into substation groups. Whilst many of these constraints are over-satisfied in practice, and could therefore be omitted without detriment to the design, preliminary elimination of constraints exposes one to the risk that the security of supply conditions will not be fully satisfied, unless the problem itself is such that a group cannot exist in the final solution. This will be the case if paths between some substations are considered to be impossible or undesirable for circuit construction. In general, if there are n_1 load substations with no direct connections to n_2 other load substations, there is no need to consider security conditions to any group composed of one or more substations from the n_1 group and one or more substations from the n_2 group.

Summarizing: if every possible circuit path is permitted, the design can usually be adequately specified by $(2^n - 1)$ lower bounded inequalities in terms of ${}^{m+n}C_2$ variables. Some reduction in these numbers can be obtained by eliminating from the start those paths along which circuit construction is either impossible or undesirable.

(5.3) Size of Network Design Solvable

A network to interconnect 9 load substations with 2 supply substations would be fully specified by $(2^9 - 1) = 511$ inequalities in terms of 55 paths. The product of the l.p. matrix row and column would be approximately 29 000 using the dual simplex method, and hence this design is within the capacity of existing computers, without external storage.

External storage systems of 48×10^6 words are understood to be available at present.[7] This would permit the solution of a 17-load-substation design with no practical limit on the number of supply substations. The computer time would probably be excessive, however, and it is considered that short cuts to a

solution must be investigated. Some possibilities are suggested in Section 8.

At present two 7-substation designs and a 9-substation design have been solved (Section 7).

(6) SPECIFICATION OF CIRCUIT CAPACITY INTO SUBSTATION GROUPS

(6.1) Distribution and Subtransmission Networks

Difficulty may arise in assessing what circuit capacity should be provided to the larger groups of substations. This is due to possible poor load sharing between a number of circuits. It may also be necessary to assume an outage of more than one circuit.

Probably only experience of load flows on networks similar to the one to be designed and a knowledge of fault statistics can enable a good assessment of the required circuit capacity to be made. One simple check is that the ratio $s/(h-1)$ should tend to fall for the group size immediately prior to each increase in number of circuits.

Alternatively there is no reason why actual substation loads should not be considered, particularly if they vary widely, provided that the group load totals are turned into equivalent numbers of circuits for writing into the constraints, as follows:

$$h = N_1 + M_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where N_1 is the first integer greater than or equal to S_L/S and M_1 is a small integer (for firm supply).

(6.2) Transmission Networks

On transmission networks, the circuit capacity into a group of substations is often assessed as the sum of two components, the planned transfer and the interconnection capacity. The planned transfer is given by the difference between the group load and the group firm generation. An empirical curve has been published[8] from which an estimate of the interconnection capacity required to a group can be obtained.

The individual circuit capacities being known and the fault risk to be guarded against having been decided, the number of circuits required can be assessed as

$$h = N_2 + M_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

where N_2 is the first integer greater than or equal to (planned transfer ± interconnection capacity)/S and M_2 is a small integer (for firm supply).

(7) EXAMPLES OF DESIGN BY THE MINIMUM-COST CRITERION

Three designs have been completed so far. The first two are described in some detail below, design and solution time figures only being given for the third.

(7.1) Example 1

It was assumed that a design was required to connect six 132/33 kV substations S_1–S_6 located at suitable points within a city to one 275/132 kV substation A established near the city's boundary [Fig. 1(*a*)], where a firm source of power was available. The average load at each 132/33 kV substation was taken as 60 MVA, except at substation S_4, where the load was assumed to be 120 MVA. It was assumed that cable circuits of 120 MVA capacity would be used on the proposed network.

The geographical disposition of the substations and estimated cost of one circuit along each path (of which there are ${}^7C_2 = 21$) are shown in Fig. 1(*a*).

The relationship between number of substations in a group and circuits required into the group was taken as:

Number of substations in group, s	Circuits required into group, h	Ratio $\frac{s}{h-1}$
1	2	
2	2	2
3	3	1·5
4	4	
5	4	1·67
6	5	1·5
7	6	1·4

It was not appreciated at the time the design equations were solved that limitation of the number of circuits into substations was unnecessary, and it was therefore also stipulated that none of the load substations should have more than four circuits connected into it, except substations S_4 into which it was said, with future requirements in mind, that eight circuits might be connected.

These stipulations led to 69 constraint inequalities.

The reduced simplex method was used and a non-integer solution [Fig. 1(*b*)] was obtained in 134 iterations. The integer solution [Fig. 1(*c*)], using the Gomory method,[5] required a further 34 iterations. The total computer time was about 4 hours.

The design was set up on an a.c. network analyser and circuit

Table 1

EXAMPLE 1: CIRCUIT LOADINGS UNDER VARIOUS OPERATING CONDITIONS

Network condition. Circuits out of commission	Circuit current % of rated current										
	Circuit										
	A–S_1	A–S_3(1)	A–S_3(2)	A–S_4	A–S_5(1)	A–S_5(2)	S_1–S_2	S_2–S_3	S_3–S_4	S_4–S_6	S_5–S_6
None ..	59	65*	65*	63	48	48	8	42	41	5	47
A–S_1 ..	0	90	90	69	51	51	50	100*	30	1	51
A–S_3(1) ..	68	0	104*	72	52	52	18	33	23	3	47
A–S_4 ..	63	83*	83*	0	60	60	12	38	78	23	73
A–S_5(1) ..	59	69	69	69	0	83*	9	41	47	17	33
S_3–S_4 ..	55	48	48	87*	57	57	5	45	0	13	63
S_5–S_6 ..	61	77	77	86*	25	25	11	39	65	50	0

* Most heavily loaded circuits.

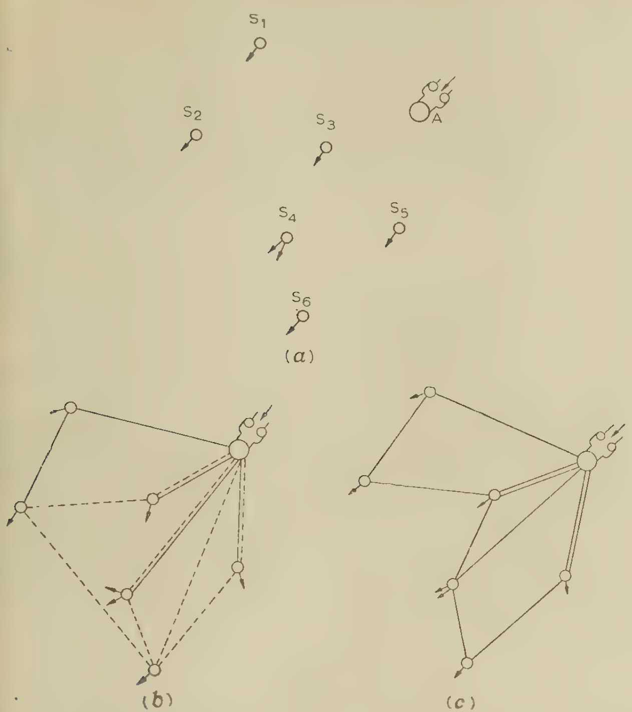

Fig. 1.—Design of 132 kV cable network.

—— 1 circuit.
---- ½ circuit.
Supply substation.
Load substation.

(*a*) Geographical disposition of substations and estimated cost of one circuit per path.

Path	Cost per circuit	Path	Cost per circuit
	£ $\times 10^3$		£ $\times 10^3$
p_{a1}	312	p_{16}	422
p_{a2}	352	p_{23}	238
p_{a3}	193	p_{24}	251
p_{a4}	299	p_{25}	365
p_{a5}	315	p_{26}	343
p_{a6}	391	p_{34}	224
p_{12}	220	p_{35}	230
p_{13}	204	p_{36}	319
p_{14}	317	p_{45}	211
p_{15}	370	p_{46}	176
		p_{56}	242

(*b*) Non-integer solution.

Path	No. of circuits	Path	No. of circuits
p_{a1}	1	p_{12}	1
p_{a3}	1·5	p_{23}	0·5
p_{a4}	1·5	p_{26}	0·5
p_{a5}	1·5	p_{46}	0·5
p_{a6}	0·5	p_{56}	0·5
All other $p_{ij} = 0$			

Cost $= f =$ £2 437 500

(*c*) Integer solution.

Path	No. of circuits	Path	No. of circuits
p_{a1}	1	p_{12}	1
p_{a3}	2	p_{23}	1
p_{a4}	1	p_{34}	1
p_{a5}	2	p_{46}	1
		p_{56}	1
All other $p_{ij} = 0$			

Cost $= f =$ £2 727 000

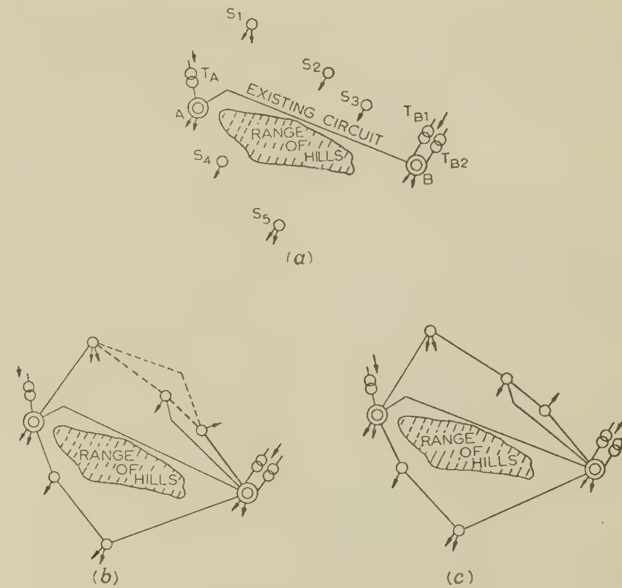

Fig. 2.—Design of 33 kV overhead-line network.

—— 1 circuit.
---- ½ circuit.
Supply substation.
Load substation.

(*a*) Geographical disposition of substations and estimated cost of one circuit per path.

Path	Cost per circuit	Path	Cost per circuit
	£ $\times 10^3$		£ $\times 10^3$
p_{ab}	82	p_{b3}	32
p_{a1}	39·5	p_{b4}	71·5
p_{a2}	51	p_{b5}	55
p_{a3}	61	p_{12}	37·5
p_{a4}	26·5	p_{13}	52
p_{a5}	53	p_{23}	23
p_{b1}	76	p_{45}	35
p_{b2}	47		

(*b*) Non-integer solution.

Path	No. of circuits	Path	No. of circuits
p_{ab}	1	p_{b5}	1
p_{a1}	1	p_{45}	1
p_{a4}	1	p_{12}	0·5
p_{b2}	1	p_{13}	0·5
p_{b3}	1	p_{23}	0·5
All other $p_{ij} = 0$			

Cost $= f =$ £291 250

(*c*) Integer solution.

Path	No. of circuits	Path	No. circuits
p_{ab}	1	p_{b5}	1
p_{a1}	1	p_{12}	1
p_{a4}	1	p_{23}	1
p_{b2}	1	p_{45}	1
p_{b3}	1		
All other $p_{ij} = 0$			

Cost $= f =$ £295 000

loadings were determined under normal and outage conditions. The results are summarized in Table 1. Owing to the low 132 kV circuit impedances, the voltage was substantially constant throughout the network.

(7.2) Example 2

It was assumed that a design was required to supply five 33/11 kV substations S_1–S_5 from two 132/33 kV supply points A and B [Fig. 2(*a*)], the area of supply being about 200 square miles. The load at substations S_2, S_3 and S_4 was taken as approximately 6 MVA and that at substations A, B, S_1 and S_5, 12 MVA. Overhead circuits rated at 20 MVA were to be used. It was assumed that two 45 MVA 132/33 kV transformers would be connected at B and one 45 MVA 132/33 kV transformer at A.

It was further assumed that a single-circuit line already existed between substations A and B, and that hilly country between substations S_1, S_2, S_3 and substations S_4, S_5 would make circuits between these two groups uneconomic.

The geographical disposition of the substations and the estimated cost of one circuit along each path (of which there are ${}^7C_2 - 6 = 15$) are given in Fig. 2(*a*).

The relationship between the number of substations in a group and circuits into the group was taken as:

Number of substations in group, s	Circuits required into group, h	Ratio $\frac{s}{h-1}$
1	2	
2	2	
3	2	3
4	3	
5	3	2·5
6	4	
7	4	2·33
8	5	
9	5	2·25

With the additional stipulation that no load substation should have more than four circuits connected into it, the problem required 48 constraint inequalities. Substations with a load of 12 MVA were treated as two 6 MVA substations.

This example illustrates points additional to those of the first example. The existence of a circuit between A and B was taken into account by including the inequality, $p_{ab} \geqslant 1$. As no circuits were assumed to be economically justified between substations S_1, S_2, S_3 and substations S_4, S_5, there was no need to include constraint inequalities to groups of the types 1, 4 or 1, 2, 4, 5, etc. Since A, although a supply substation, was assumed to have only one infeeding transformer, it was necessary to include inequalities specifying minimum numbers of circuits to groups containing substation A. The minimum numbers of circuits to such groups would be one less than the Table above indicates, however, since the 132/33 kV transformer provides one circuit.

The non-integer solution [Fig. 2(*b*)] was first obtained using the reduced simplex method. This required 69 iterations with a computer time of just over one hour. The problem was later solved using the dual simplex method, when the non-integer solution was obtained after 15 iterations in a little under ten minutes.

The integer solution [Fig. 2(*c*)] required two further iterations from the non-integer solution.

This design, with one 132 kV overhead line circuit between the 132 kV busbars at substations A and B, was set up on an a.c. network analyser, and circuit loadings and busbar voltages were determined under normal and outage conditions. The lowest voltage recorded was 91% at substation S_4, with the 132/33 kV transformer at substation A out of commission. This was well within the tapping range of the 33/11 kV transformer. The circuit loadings are summarized in Table 2.

(7.3) Example 3

A network was required to connect eight substations with widely varying loads and some with a net surplus of generation to a ninth substation. The circuit capacity already connected to this substation was such that it could make up any deficit required to the other eight substations and hence it was treated as a supply substation when writing the constraint inequalities. Of the 36 possible paths between substations, only 23 were assumed to be suitable for the provision of circuits. In order to be sure that adequate capacity would be provided for generation connected to the network, minimum circuit capacities had to be specified to all possible groups of substations, even those with a net surplus of generation. The circuit capacities were estimated from eqn. (13).

With the stipulation that one circuit already existed between two of the substations, the problem required 199 lower-bounded inequalities in terms of 23 paths.

Two designs were obtained, with different security conditions, in each case the dual simplex method being used. One design was completed in 58 iterations and the other in 79 iterations, with computer times of about 1½ hours and 2¼ hours, respectively.

Table 2

EXAMPLE 2: CIRCUIT LOADINGS UNDER VARIOUS OPERATING CONDITIONS

Network condition	Circuit current % of rated current — Circuit											
	T_A	T_{B1}	T_{B2}	A–B	A–S_1	A–S_4	B–S_2	B–S_3	B–S_5	S_1–S_2	S_2–S_3	S_4–S_5
Normal (*a*)	62*	44	44	14	41	48	35	47	45	23	17	19
Normal (*b*)	57*	47	47	17	35	43	38	49	50	26	19	12
T_A out of commission (*a*)	0	77	77	65	8	8	61	72	90*	72	42	25
T_{B1} out of commission (*a*)	71	79*	0	4	47	54	32	43	53	14	13	23
A–S_1 out of commission (*a*)	52	49	49	10	0	57	58	69*	36	66	39	27
B–S_2 out of commission (*a*)	65	43	43	17	52	45	0	71*	47	12	41	16
B–S_5 out of commission (*a*)	77	38	38	26	35	95*	40	51	0	27	21	65

(*a*) No through power transfer between 132 kV busbars at substations A and B.
(*b*) Through power transfer of 27 MW, 13 MVAr from 132 kV busbar at B to 132kV busbar at A.

(8) FURTHER DEVELOPMENT OF THE MINIMUM-COST DESIGN METHOD

It will have been obvious from the foregoing that the major difficulty in applying this method is due to the large number of constraint inequalities needed to specify a design. Even with the computer storage problem resolved, the writing of the constraints would quickly become extremely tedious, if not impossible, and their solution time impractical.

Some ways in which these difficulties might be minimized are suggested below.

(8.1) Use of Special Computer Programmes

(8.1.1) In the Preparation of the Design Equations.

Once the number of circuits required to be connected into groups of substations of various sizes or load totals has been decided, the writing of the constraint inequalities is a purely mechanical process. It has already been shown that the solution of a problem of any size requires the use of a computer, and there seems no reason why the writing of the constraints should not itself be done by the computer. The saving in time and reduction of possible errors in coding the large matrices for computer solution would also be significant.

If this were done, the presentation of a network-design problem to a computer would consist of a list of the substations (with perhaps load and generation data for each), estimated cost of one circuit along each possible path and a tabulation of numbers of circuits required into substations groups of the various sizes or load totals possible.

(8.1.2) In the Solution of the Design Equations.

It may have been noticed that, even in the non-integer solutions, only integer or integer plus half-circuits appears. In fact, the matrix elements in all the iterations have been observed to be either small integers or fractions with small dividers. It is possible that a computer programme could be devised to take advantage of this empirical observation, thereby increasing the effective storage by some small multiple.

Contrary to what might be expected, the proportion of zeros in the initial matrix does not appear to increase as the number of substations increases. This proportion is given by

$$1 - \frac{\sum_{l=1}^{n} {}^{n}C_l(m+n-l)}{{}^{m+n}C_2(2^n-1)} \quad . \quad . \quad . \quad . \quad (14)$$

and over the range $m = 1$, $n = 4$–14, is approximately 50%. It is doubtful, therefore, whether any attempt to pack non-zero elements in the computer store would be worth while.

(8.2) Reduction in the Design-Matrix Size

It has already been shown that elimination of paths along which circuits are impossible or undesirable will effect some saving in the number of constraints. This approach could be extended by not including those paths along which circuits appeared by intuition to be most unlikely. It is doubted whether significant reductions in the matrix size could be obtained by this means. In Example 3, a reduction of 36% in possible paths gave a reduction of only 22% in constraint inequalities.

A more promising approach is thought to be the use of Kron's 'tearing' technique.[9] A statement is made that adoption of this not only enables larger systems to be optimized but also reduces numerical computation.

Alternatively, sampling of the constraint inequalities might be possible. When the solutions obtained in the examples were substituted in the initial design inequalities, it was found that about 80% of these were over-satisfied and could therefore have been omitted with no detriment to the designs. It is suggested then that 30–40% of the design inequalities should be selected and a design obtained from these. The ones chosen would include all the single-substation constraints, constraints for two adjacent substations, and generally constraints for the more closely sited substations. An intuitive design might also be used as a basis for sampling constraints. The non-integer solution obtained from the sample of constraints would be substituted in all the constraints. Any that were not satisfied would be taken into the sample, at the same time those well over-satisfied being dropped out. A new non-integer solution would be obtained and the process repeated until a design satisfying all the constraints resulted.

This method would 'fail to safety' as far as cost was concerned, since, in any linear programme, a reduction of constraints tends to decrease the minimum cost obtained.

(8.3) Reduction of Solution Time

A fairly near optimum, if not optimum, network design can be obtained by inspection. Looked at from the l.p. aspect, if this solution could be used as a first feasible solution it should very materially reduce the number of iterations required to obtain the minimum-cost design.

One method of doing this is described by Churchman, Ackoff and Arnoff.[10] As the method appears to involve the solution of a number of sets of simultaneous equations, further work would be necessary to determine its feasibility when applied to a large example.

(9) NETWORK DESIGN BY OTHER CRITERIA

Two other design criteria have been investigated. A network with a minimum total apparent-power by distance product is given by one, and a network with a minimum total circuit length by the other.

(9.1) Design by the Minimum Apparent-Power by Distance Product

Assume there are a number of supply substations $\mathrm{S}_1, \mathrm{S}_2, \ldots \mathrm{S}_m$, at which apparent powers $S_1, S_2, \ldots S_m$ are available and a number of load substations $\mathrm{S}_n, \mathrm{S}_l, \ldots \mathrm{S}_r$, at which loads S_n, $S_l, \ldots S_r$ have to be supplied. If S_{ij} is the assumed apparent-power transfer between substations S_i and S_j, the application of Kirchhoff's first law at each substation gives the following sets of equations:

$$\sum_{i=1 \neq s}^{r} S_{is} \leqslant S_s \quad (1 \leqslant s \leqslant m) \quad . \quad . \quad . \quad (15)$$

$$\sum_{i=1 \neq l}^{r} S_{il} = S_l \quad (n \leqslant l \leqslant r) \quad . \quad . \quad . \quad (16)$$

The apparent-power by distance product for the network will be

$$g = \sum_{i=1 \neq j}^{r} \sum_{j=1 \neq i}^{r} S_{ij} l_{ij} \quad . \quad . \quad . \quad . \quad (17)$$

which can be minimized subject to eqns. (15) and (16) as constraints.

The product $S_{ij}l_{ij}$ is a measure of the weight of conductor required between S_i and S_j, and if circuit costs were proportional to conductor weight, this criterion would give a minimum cost design. As this is not so, it cannot be relied upon and only gives a minimum cost connection for certain geographical configurations of substations.

(9.2) Design by a Minimum-Circuit-Length Criterion

It may be necessary on some occasions to design a network to interconnect a number of points so that each point has at least one connection and the total circuit length is as small as possible.

This is practically the design condition for an h.v. rural network or an m.v. cable network.

To interconnect r points $(r-1)$ circuits are required, and hence to specify at least one circuit into each point the following inequalities can be written down:

$$\sum_{i=1\neq j}^{r} p_{ij} \geqslant 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

$$\sum_{i=1\neq j}^{r} \sum_{j=1\neq i}^{r} p_{ij} = r-1 \quad . \quad . \quad . \quad . \quad (19)$$

$$p_{ij} \leqslant 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

The length of interconnecting circuits to be minimized will be

$$f = \sum_{i=1\neq j}^{r} \sum_{j=1\neq i}^{r} l_{ij}p_{ij} \quad . \quad . \quad . \quad . \quad . \quad (21)$$

The same solution can be obtained with considerably less effort by choosing circuits in ascending order of length, omitting any circuit which completes a loop, until all the points are connected.

(10) CONCLUSIONS

Designs produced by the method outlined in Sections 4–8 can only use circuits along those paths permitted when writing the design equations, and only incorporate the system design engineer's initial estimate of the numbers of circuits required into substation groups. Even with all inter-substation paths permitted, one may still be influencing the design, as the T-feeder arrangement so common in transformer-feeder network design has not been provided for. Again, so far only single-voltage network design has been attempted, and this has meant that the network supply points have had to be nominated at the beginning of the design. It is considered that the method could be extended to 2-voltage-level design and to permit the possibility of T-feeders.

Poor load sharing between circuits may invalidate otherwise reasonable estimates of required circuit capacity, particularly into the larger groups of substations. As it happened, the group-circuit estimates used in the examples were proved to have been satisfactory by the network analyser studies. However, the l.p. design method cannot in itself make allowance for poor circuit utilization, and it is therefore necessary to carry out load-flow studies on any l.p. design. It will also be necessary to check fault levels unless it is known that the supply-point arrangements are such that the rupturing capacity of the switchgear proposed cannot be exceeded, e.g. the use of three 132/33 kV 45 MVA transformers to supply a 33 kV network equipped with 750 MVA switchgear, as in Example 2 (Section 7.2).

Hence it is not suggested that adoption of this technique would make unnecessary the present tools of the system design engineer —the network analyser and the digital computer as an instrument for network analysis. Nor would it necessarily lead to economies on any single reinforcement. However, about £100 million is invested yearly in extensions to the transmission and distribution networks in Britain alone, and any means of achieving even a marginal saving on this would repay further investigation.

(11) ACKNOWLEDGMENTS

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DISCUSSION ON THE ABOVE TWO PAPERS BEFORE THE SUPPLY SECTION 10TH FEBRUARY, 1960

Mr. T. R. Warren: The papers show how the digital computer can be used to help to solve our system design problems in both transmission and distribution fields.

The paper by Mr. Knight shows how its aid can be enlisted to solve a wide range of problems of the type more commonly encountered in transmission and h.v. distribution. To this extent the author has performed a useful service, but the examples he gives rather emphasize that gaps still exist in our knowledge of some of the fundamental principles which underlie much of this work, and there is a danger that traditions in system design might become established before they have undergone a sufficiently searching investigation.

For example, in Fig. 1 the author does not appear to have examined the possibility of using duplicate transformer feeders supplying pairs of substations, thus eliminating 132 kV switchgear everywhere except at the supply point. Then, again, did the author consider whether it would be feasible to distribute more load at 33 kV from substations supplied at 275 kV? Examination of a similar problem in Glasgow showed that this was, in fact, the cheapest solution in that particular case.

One of the prices we have to pay for the introduction of each new transmission voltage is the need to introduce an additional voltage transformation between the alternator terminals and consumers' terminals. By about 1970 the bulk of the energy

generated in Great Britain will undergo five such transformations, and it is necessary to consider carefully the conditions under which it is legitimate to leapfrog from one voltage to the next but one. When the answers to these questions have been given we shall be in a better position to follow the lead given by the author with greater assurance that we are, in fact, achieving the logical design of electrical networks which he promises in the title of his paper.

In the paper by Dr. Grimsdale and Mr. Sinclare the authors take as the starting-point the principles governing the design of distribution systems for supplying housing estates which were first put forward in the report referred to in Reference 2. Since I was concerned with the preparation of this report, in which I had the assistance of a small committee of expert system design engineers, and since I provided the theoretical background on which it was largely based, I was naturally most interested in the methods devised by the authors for pinpointing substation positions and for the selection of the optimum routes and sizes for the distributors.

When the report was first produced, a few engineers expressed doubts concerning our advocacy of what we termed the tapering of cables and the employment, where necessary, in any one scheme of all the standard cable sizes available. It was felt that a 'tailor-made' scheme would not lend itself so readily to reinforcement to meet increased loads as would a completely interconnected system built up with cables of large size.

At the Vesting Date, however, many schemes were costing £90 per house compared with the corresponding figure to-day of approximately £50 per house for an a.d.m.d. of 3 kW per consumer, and the authors have now shown how computer techniques can be employed to effect further savings in cost. With the large number of floor-warmed housing estates now being built it is necessary to allow for a demand of 6 or 8 kW per consumer, and in one case the estimated demand was 15 kW per consumer.

Mr. J. G. Miles: It will generally be agreed that both papers are of considerable significance to the system engineer, since they consider, almost for the first time, the application of large digital machines to the actual design of networks, as distinct from specific items of analysis. In the paper by Mr. Knight the linear programming process used is a true synthetic method, whereas the automatic methods normally used for apparatus design rely principally on adding automatic self-correcting features to fairly standard design procedures.

The author's linear programming process is obviously restricted in that the restraints which form its basis must be linear, so that its immediate usefulness is limited to medium-voltage fairly highly interconnected networks, where voltage control and stability limitations are not normally a limiting feature, and in the process it is inherently assumed that all circuits are capable of supplying their rated apparent power. It is tempting to speculate, however, on how such a procedure could be incorporated into an overall automatic design programme for high-voltage networks.

The programme would probably be of the following form: Starting with assumed generation and load data, path lengths between all stations and cost per circuit along those path lengths, a tentative voltage and conductor size would be chosen, and the circuit thermal ratings and required number of circuits to the various substation groups calculated. A linear programming process would then lead to an initial estimate of the optimum number of circuits per path. It might be necessary at this stage to check approximately for steady-state stability, at any rate on the longer paths. A load-flow calculation would then check for overloaded circuits or substation voltages outside specified limits. The presence of these would cause modification of the initial connection scheme and/or automatic determination of the necessary voltage-correction equipment. Characteristics would then be assumed for generators and switchgear and the overall stability would be checked on that basis. If the network were limited in stability it would be necessary either to modify the generator or switchgear characteristics or to remodify the initial connection scheme. Finally, on realizing a technically satisfactory scheme, the programme could calculate fault levels and losses and work out an overall capital cost. This could be repeated for different assumed voltages, and it might be necessary to carry out subsidiary economic comparisons, say between the modification of generator characteristics and the number of circuits per path.

It is possible that the number of modifications which it would be necessary to make to the initial scheme because of additional restrictions might make it difficult to justify spending large amounts of computer time and money on the initial work. This could only be assessed by trial, but, in any case, the methods described will undoubtedly make us consider more carefully the possibilities of an overall automatic design.

Mr. J. L. Egginton: The electronic computer is a powerful tool, being fast, accurate and tireless, but it has no intelligence and extreme care is necessary in accepting the results. The authors feel that human intuition should be eliminated, but this can lead to difficulties. The computer is a new tool which enthusiasts tend to use for jobs which could be done more simply, quickly, and as accurately, without it. This leaves many problems for which the computer is ideal.

In the paper by Dr. Grimsdale and Mr. Sinclare there are cases where the computer has erred; where it has been used unnecessarily; and where it is, in fact, the best tool for the purpose.

The machine erred by accepting a formula without considering its implications. It has calculated correctly that five substations are required, but any intelligent engineer would then consider the total transformer capacity. Table A shows that there is no

Table A

No. of substations, N	5	6
Apparent power per substation, kVA	347	289
Standard transformer apparent power, kVA	500	300
Incremental cost per transformer, £ . .	600	360
Constant cost per substation, £ . .	500	500
Total cost per substation, £	1 100	860
Total cost of N substations, £ . .	5 500	5 460

significant difference in cost between five and six substations, although with six we should save about 20% on distribution mains. The computer has worked correctly to the formula, but no engineer would overlook the possibility of six substations being cheaper.

Fig. A shows the boundary of the estate and substation positions obtained by fitting to the plan five paper discs each equal to the area supplied by one substation. This took seven minutes. Four of the discs are close to the authors' positions. The fifth is in the playing field (an unsuitable position) and it would be moved to the school, where there is a load of 150 kW. Having fixed the substation sites, the engineer then tries a large number of possible routes and conductor sizes. This is an excellent job for the computer and engineers who have the task will be delighted that it can be done for them, thus saving much laborious calculation.

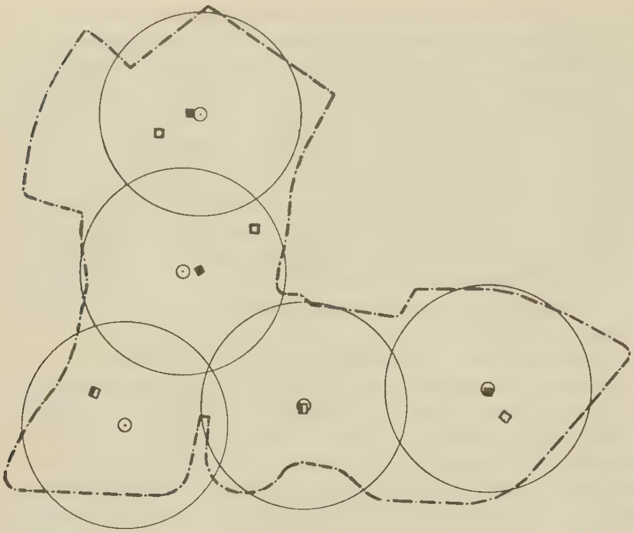

Fig. A

Estate boundary.

Substation positions

Actual.
Calculated (optimum feasible site).
Coincident.
Centres of circles of 565 ft radius.

Mr. J. H. Wensley: I should like to know something about the cases in which the methods broke down. The problems dealt with are fairly rational, but what happens in the irrational cases? Let us take a housing estate on the lines of the pre-war long ribbon development. If we start fitting squares to this and we have previously decided that we want three transformers we can fit them in quite easily. Large squares will not be suitable because they are too 'fat' for this ribbon development, but we can probably have about 100 smaller squares in a long string, and then we shall have no idea where to put the transformers. That is admittedly an extreme case, but we might have an estate consisting of a large bulk with a limb or two limbs attached to it. I do not think that the computer technique outlined in the paper by Dr. Grimsdale and Mr. Sinclare will cope with that problem, although iteration from the boundaries might deal with it.

The paper by Mr. Knight fills this gap to a certain extent in dealing with the maximum size possible using existing computers. It would appear to be the sort of limitation to be placed on this method. I am interested in seeing whether the techniques produce subsidiary information which can be used at a later stage, and particularly on the stability investigation for the network. I think that a step-by-step technique using Kron's method will make it possible to arrive at the required solution and, en route, obtain a great deal of information which will be needed to investigate the dynamic stability and also the protection of the circuits in case of failure.

Mr. H. J. Sheppard: I propose to limit my remarks to the paper by Dr. Grimsdale and Mr. Sinclare. The Chief Engineer of the Yorkshire Electricity Board in suggesting the investigation which forms the subject of the paper had no strong expectation that it would lead to the adoption of a method on the lines of that described. It seemed worth while, however, as a check on existing methods for designing housing-estate distribution systems, having regard to the large capital expenditure involved.

It is desirable to make some appraisal of the results achieved, and I would draw attention to the cost comparisons in Table 1. Referring to cols. 3 and 4, the adoption of calculated instead of actual substation sites would in some instances (e.g. substations Nos. 1 and 5 in estate No. 2) be likely to necessitate buildings instead of outdoor equipment, with consequently increased costs of perhaps £400 per substation. Thus the savings on cabling costs in col. 4 might well be outweighed by increases in other items.

I feel sure that the 'excess facilities' shown in Fig. 5 (such as the additional m.v. cable outlets from substation No. 3 and the heavier interconnection between substations Nos. 1 and 2) would in practice be provided, so that the savings in cost would be as shown in col. 5 rather than col. 3 or 4 of Table 1. The manual design for estate No. 2 received special attention and is undoubtedly better than average. The mean saving on the other three schemes is about 4% of the cost of m.v. cable.

At the present rate of connection of the larger new housing estates in the Yorkshire Board's area, this would represent a saving in capital expenditure of about £10 000 each year, and a total of perhaps ten times as much for the whole of England and Wales. It is doubtful whether that saving is sufficient to justify the provision of computer facilities and the necessary organization, unless they can be profitably employed for other aspects of the Area Boards' engineering work.

Mr. T. W. Berrie: I am optimistic about these computer techniques, and also realistic.

In Mr. Knight's paper the first use, and an immediate one, is to supply a template by which to judge electrification schemes. There are various ways of doing this: for example, we can look back over the last ten years, starting with what was there ten years ago, applying the loads which we have to-day and examining what we get by this technique compared with what we have to-day; or we can divide the scheme into stages and see whether or not we have been uniform with the same basic standards of security and types of design in each stage; or, again, we can examine various parts of the network to see whether the same uniform assumptions have been made in each case.

With regard to the future this is a problem of synthesis plus analysis, and we cannot separate one from the other. I would, however, give some prominence to synthesis, because here we can kill two birds with one stone. This is because, in order to make the method realistic, we must increase the number of substations which can be dealt with, even if it means making an intuitive guess at the beginning or sampling the equations.

Having obtained the first design by synthesis, we can analyse it, and then, having found out the weak points, return to the original equations of constraints and take a different sample, bearing in mind the result of the analysis.

We must also eliminate the linearity in transmission work, especially with regard to double-circuit lines.

Finally, is the technique in Mr. Knight's paper more suitable for transmission or distribution? I began with a strong bias towards distribution problems, but, having tried it for transmission on a limited scale, I am not at all sure of this and only the future will tell.

Mr. D. G. Taylor: I propose to confine my remarks to the paper by Mr. Knight. The author states that this is a topological problem of finding the cheapest acceptable circuit configuration between a given set of substations. There is, in fact, a finite number of possible configurations for a given number of substations if the number of circuits per path is restricted to no more than two; this means that, in principle, one could make a very simple programme in which all possible configurations were generated and their cost evaluated, punching out or displaying the result whenever the cheapest so far had been found. In many problems of this kind the method is a powerful one, particularly if the problem is non-linear.

In terms of the simplest problem that the author describes (see Fig. 1 of the paper), in which there are six load substations and

one supply substation, there are 21 possible paths, and, since each path can have 0, 1 or 2 circuits, there are 3^{21} or about one million possible configurations. On medium-size computers, with about 10 000 words of storage available, it will take several (probably less than 10) hours to run through the one million possibilities involved. However, if a method can be found for generating, not all possible configurations, but only those which satisfy the requirements of the problem this time might be reduced appreciably. Something of this kind has been done in the context of switching problems, where it is possible in a routine manner to generate all possible circuit configurations between given nodes which provide a path between two particular nodes in a circuit. Certainly a method of this kind would make very slight demands on computer storage.

Mr. J. I. Bird: Both papers are concerned with the mathematical design of electrical power systems, but the problems are entirely different. The design of the housing-estate distribution system is based on a reasonably firm knowledge of the geographical area to be supplied and the ultimate maximum demand. It can, in fact, be treated as an entity separate from the remainder of the supply network. This is certainly not so in the problem discussed in the paper by Mr. Knight. He demonstrates a method of obtaining mathematically the minimum-cost design of a section of an interconnected transmission network, and, in my view, this cannot readily be determined except in the context of the development of the system as a whole. In system planning, account must be taken of possible future developments at a higher voltage and existing assets at all voltages, as these usually have a very considerable bearing on the choice of the immediate scheme. The cheapest development of a small section of the network considered in isolation may not fit in with the optimum development of the system, and it is therefore necessary always to have regard to this fact in the development of the sub-transmission element.

It is appropriate to mention a further application of linear programming in respect of power-system planning with which I am concerned at the moment. This concerns the integration of new power stations into the system. Apart from detailed site considerations, it is possible to express all the relevant data, i.e. cost of generation and energy transport, in the form of linear equations which could be solved by methods similar to those described in the paper by Mr. Knight. The Deuce computer available would allow only a relatively small problem to be solved by this method. However, it has been found possible to express the data in the special form of a transportation problem which enables a much larger problem to be solved, and it is practicable to consider the C.E.G.B. system as a whole.

I agree with the authors that linear programming enables a logical rather than an intuitive solution to be obtained, and as larger and faster computers become available, it should be possible to perfect and extend the technique.

Mr. C. T. Stubbs: I propose to confine my remarks to the paper by Dr. Grimsdale and Mr. Sinclare and, in particular, to one aspect of it, namely that dealing with consumer diversity. It is a fundamental of the design of the tapered radial system of distribution that factors for the unbalance prevailing in the l.v. distributors and also for the consumer diversity must be established in advance. In this connection the authors mention that they have taken those assumed in Reference 2 of the paper.

In using the formula for diversity to which they refer, they are adopting one which I think has been used by all sponsors of the tapered radial system over the last ten years or so. The formula, which determines the total load on a l.v. distributor as $(3n + 8)$ kilowatts where n is the number of consumers supplied from the distributor, was first referred to in a B.E.A. Utilisation Report.

The basis of the formula was a test which had been undertaken on a small estate of 87 prefabricated dwellings in south-west London, where the facilities provided consisted of a cooker, a solid-fuel fire in the main living room, which also served the hot-water system, one 2 kW fire in the principal bedroom and a 2 kW immersion heater to supplement the hot-water supply. The test was conducted over a period of 10 or 11 days at a time when the midday temperature never dropped below 40° F.

It is curious, therefore, that despite a steadily developing use of electricity for space heating, networks are still being planned on a load pattern evolved 13 years ago on an estate where the electrical equipment was so very inadequate judged by modern standards.

The authors suggest that it is economically unjustified to install a completely interconnected system, but in the circumstances can it be said to be sound engineering practice not to do so?

Dr. P. D. Aylett: In the past I have been concerned with the design of high-voltage transmission networks, and I always considered that the designs accepted appeared to rely rather too much on the preferences, or perhaps the prejudices, of individual engineers.

The department in which I worked produced high-voltage transmission schemes of various kinds, costing about £30 million per year. In each case only one or two alternatives were considered, and even the saving of a few per cent on these large sums is obviously worth while. I think that the objective assistance which might be offered by these new mathematical methods used by the authors is greatly to be desired.

Are we witnessing in these papers isolated eccentricities, or is this the first swallow which heralds the summer? There is no doubt in my mind that the use of large computers, not only in electrical engineering but in all fields, is, over the next 10–15 years, going to transform engineering design. We must certainly see that students now in training are able to become familiar with the use of digital computers, because the designers of the future will use these powerful tools.

These papers are pioneering efforts, and we should not be too critical of them in detail.

Looking at the general system design problem, which is covered in Mr. Knight's paper, the most important factor missing is time. He has dealt with the spatial aspect, but, in fact, the design of electrical networks is an investment problem. How much shall be spent this year or on this occasion? To what extent should excess facilities be provided for future load growth? These kinds of problems are covered by the mathematical technique of dynamic programming. If we consider the first network in the paper, it seems quite possible, taking into account the whole life of the network, the loads arising in the future and the possibilities of future development, that some excess facilities should have been installed, and perhaps the first network configuration given in terms of $1\frac{1}{2}$ feeders in various places should have had two feeders when we take into account the need to minimize the investment per year over a period of 10–20 years.

Mr. H. B. Dreyfus: The paper by Dr. Grimsdale and Mr. Sinclare deals with a very definite scheme for a new and compact housing estate where the loads are known or can be predicted and allowance can be made for the future. The paper by Mr. Knight, on the other hand, is concerned with an indefinite scheme, and, as Dr. Aylett states, the time element has been left out. It was a growing scheme and did not start with a number of 60 MVA supply points; probably it had only one to begin with and then grew.

Considering the definite scheme, the paper by Dr. Grimsdale and Mr. Sinclare shows how accurate the original design engineers were. The point made by the authors, that their

scheme showed a smaller number of cables coming out of each substation, is correct, but it ignores the question of security of supply; possibly the original engineers put in more cables for that purpose.

In Mr. Knight's paper, the technique described is very useful as an occasional check that the design engineer is doing his job accurately. I admire the way in which the author has written into his programme how to overcome the wayleave difficulty, but cannot a programme be written which states that lines must be whole numbers, or does that require too large a computer?

Has the author carried out any basic design? That is where the technique can be most useful. We all know the arguments about connecting two substations, i.e. two separate supply points, designing them either as an interconnected system or as radial feeders not interconnected. Has the author investigated this problem to see which is the more economic, both from the point of view of starting from scratch and starting from either system and then reinforcing it, in order to find which is the more economic, including incidence of expenditure in the final comparison.

Mr. G. S. Buckingham: I think that the tools described in th paper are going to enable us to design our future distributio and transmission networks much more accurately and consis tently. We should not be too critical of what is being done It may be true that good designers, using their intuition as wel as their skill, will for some time be able to compete with thes non-intelligent machines, but it will not be long before com plicated networks will require their assistance. We shoul encourage their use in every way from now on.

I would support Dr. Aylett in his reference to investmen policy. Present designs are based on an a.d.m.d. of 3 kW Nowadays, with floor warming developing rapidly it is possibl to have a figure as high as 15 kW; all floor-warming circuit are time-switch controlled and can come on together withou diversity. It is therefore necessary to sound a warning i digital computers are to be used to produce minimum design very close to average present-day requirements. It is importan that our investment policy should include a certain amount o 'fat' in our system, so that we may provide some future capacit for posterity in the same way as our forerunners did for us.

THE AUTHORS' REPLIES TO THE ABOVE DISCUSSION

Dr. R. L. Grimsdale and **Mr. P. H. Sinclare** (*in reply*): In reply to Mr. Egginton, the method does, in fact, include tests for different numbers of substations, as can be seen from Fig. 4. For the estate in question, the case of five substations is taken in order that a comparison can be made with the expertly designed network which also used five substations.

The method of choosing the substation sites suggested by Mr. Egginton does not appear to take into account the possibility of non-uniform load density and would not be satisfactory for certain estate shapes.

The ribbon-development type of estate referred to by Mr. Wensley is interesting and, in fact, led to the use of the boundary-division method for choosing the starting-points. The square-fitting method would probably work in most cases, but under these circumstances the final size of the squares would be small and they might be all at one end. However, the iterative process would then distribute the sites uniformly.

We agree with Mr. Sheppard that it would be necessary to employ a computer on other work besides estate design. If for no other reason, the time used for estate designs for the whole of England and Wales using a computer like the Mercury would be less than five weeks per year, and using a faster machine under construction, the total time would be about one day. The savings obtained in this time would make a valuable contribution towards the operating expenses of the computer.

The choice of a suitable a.d.m.d. is a matter of engineering policy. The particular value chosen in the example is the same as that used when the estate was manually designed. It is a trivial matter to change this value as it is only a parameter of the programme. Indeed, it is very easy to estimate the extra cost of using an a.d.m.d. in excess of established practice. The provision of extra outlets from substations can be dealt with in the same way.

Mr. U. G. W. Knight (*in reply*): Mr. Warren has pinpointed one of the present weaknesses of the suggested method, i.e. the basic design philosophy is, in practice, fixed when the design equations are written. Thus the possibility of teed transformers was not permitted (unless the tee points were at one of the load points) nor was the possibility of a design with the omission of one voltage transformation. If one assumes that a mixed network (e.g. 275/132/33 kV and 275/33 kV) is not desirable the problem is then of finding the cheapest way to satisfy the system requirements for the alternative design methods and not of obtaining a design with every possibility permitted initiall This makes the problem easier and would, in fact, be a solutio on the lines suggested by Mr. Miles, who proposes a compre hensive comparison of optimum schemes each based on a di ferent set of tentative parameters.

Even so, this, or the basic design work suggested by M Dreyfus, requires the writing of design equations for a 2-voltage level network, and this seems considerably more difficult tha the examples quoted. This is because one is then invariabl dealing with circuits of differing capacities. Take, for instance the design equations necessary for a proposed network to suppl a number of 33 kV load points with loads between, say, 10 an 25 MVA from a supply source by a network of 132 kV circuits 45 MVA 132/33 kV transformers and 20 MVA 33 kV circuits For a load (at one substation or a group of substations) o 25 MVA and with the frequently accepted standards of security the minimum 33 kV cable/transformer combinations permissibl would be as follows:

Number of cables	Number of transformers
3	0
2	1
1	2
0	2

This can be formulated algebraically provided that one can sa where the first transformer (if any are to be provided at all t this particular substation or group) is to be sited, which is n difficulty for the single substation constraint but appears t involve an arbitrary singling out of one substation from a grou of substations.

Mr. Egginton states that a computer has no intelligence neither has it, and this perhaps amounts to the same thing i many cases, any appreciable past experience. I have felt tha the method is very 'heavy handed' in that every possibilit within the initial problem statement is permitted. On the oth hand, a system design engineer has an overall perception of th problem and much past experience which enables him to produc a near-optimum or optimum network topography at sho notice but also an inability to produce more than a limite number of solutions.

In reply to Mr. Wensley the irrational case should not caus difficulty, again since every possible system of connections permitted. I do not think very much subsidiary informatio

can be obtained during the actual solution of the design equations. On the other hand, analysis of a computer design should produce information of value in subsequent designs.

Mr. Berrie's suggestion that the method provides a template against which to judge schemes is valuable. The checking of numbers of circuits into every possible group of substations is obviously an alternative, although somewhat approximate, to a load-flow solution when examining an existing network or design. The non-linearity of cost and number of circuits with double-circuit lines can, it is hoped, be tackled in two ways. Constraints can be added which will ensure either 0, 2, 4, etc., circuits between pairs of substations, and the problem remains a linear programme for which an integer solution can be obtained. Alternatively, the function can be written

$$f = \Sigma c_{ij} p_{ij} \left(\frac{2a(2 - p_{ij}) + p_{ij} - 1}{2a} \right)$$

where c_{ij} is the cost of a d.c. 1 line, a is the ratio of the cost of a d.c. 1 line to a d.c. 2 line and $p_{ij} \leqslant 2$. This is now a quadratic programme. A non-integer solution can be obtained by known methods, but experience on another problem has shown that the storage difficulty is greatly increased.

Mr. Taylor's *ad hoc* approach is interesting and may well offer a worth-while alternative. I imagine that, if a programme for generating the feasible configurations were available, extension to, say, 2-voltage-level design would be easier than with the l.p. method.

Mr. Bird comments on the desirability of considering the local development in the context of the whole system and of existing assets. This broader consideration might well resolve into a comparison of a small number of optimum designs, each based on different sets of power sources or alternatively a 2-voltage-level design. One of the difficulties with the inclusion of existing assets is the writing of equations which permit the diversion of an existing circuit into a new substation. This is possible as shown in Fig. B, where an artificial substation is

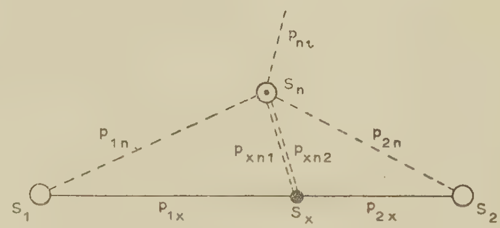

Fig. B.—Extensions to existing network.

—— Existing circuit.
---- Paths for new circuits.
○ Existing substation.
⊙ New substation.
● Artificial substation.

$$p_{1x} = 1 \qquad p_{xn1} \leqslant 1$$
$$p_{2x} = 1 \qquad p_{xn2} \leqslant 1$$
$$p_{xn1} = p_{xn2}$$
$$= \ldots + c_{1n}p_{1n} + c_{2n}p_{2n} + c_{xn1}p_{xn1} + c_{xn2}p_{xn2} + c_{ni}p_{ni} + \ldots$$

assumed at a point on an existing circuit run with the best access to a new substation.

Dr. Aylett and Messrs. Dreyfus and Buckingham mention the importance of including the incidence of expenditure. In the paper I adopted the view that the design is required to meet a specific set of estimated load conditions. Rather than attempt to formulate the problem with variable loads, and as an alternative to dynamic programming, I think again that a comparison of a number of solutions based on specific intermediate sets of loads might be of value.

DISCUSSION ON

'A METHOD OF MEASURING SELF-INDUCTANCES APPLICABLE TO LARGE ELECTRICAL MACHINES'*

Before a Joint Meeting of the MEASUREMENT AND CONTROL, SUPPLY *and* UTILIZATION SECTIONS *5th January,* 1960.

Mr. D. H. Thomas: From an educationalist's viewpoint the paper is of great fundamental interest, and I hope that experiments based on this method of measurement will be incorporated in laboratory teaching. Fig. 6 showing an inductance variation with time is initially confusing until it is realized that the true independent variable is current, and that the current varies with time. I would like to have more details of the timing device for calibration, mentioned in Section 2.2.

Mr. S. Rudzinski: I would like to describe briefly a method of measuring the in-phase and quadrature components of an admittance over a certain frequency range. As the admittance components are determined directly, and not as a function of the admittance angle θ, the percentage accuracy of the measurements is maintained even when $\cos\theta$ or $\sin\theta$ approach zero.

The apparatus, developed at Imperial College under the supervision of Dr. Morris, was used to obtain frequency-response curves of a machine, from which an equivalent circuit of the machine was subsequently synthesized. The method, illustrated in Fig. A, employs a null technique, and consists essentially in

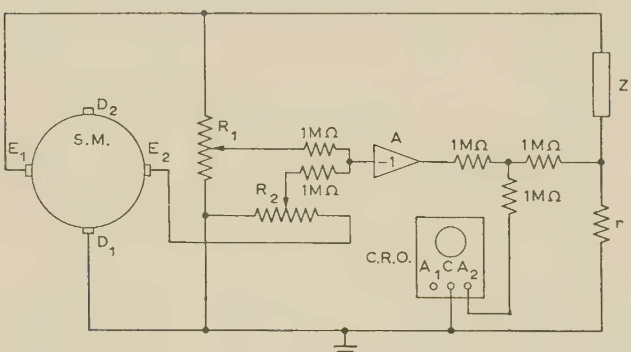

Fig. A.—Illustrating a method of measuring the in-phase and quadrature components of an impedance at variable frequency.

SM. Schrage machine.
R_1, R_2 10-kilohm 4-decade resistance boxes.
A. Amplifier of unity gain.
r. A standard resistor.
Z. Unknown impedance.

balancing two variable-magnitude voltages, mutually displaced by 90 electrical degrees, against the voltage drop across a known resistance *r*. The 2-phase supply is obtained from three brushes of a 2-phase Schrage machine as described elsewhere.† The magnitude of each brush voltage is constant and independent of frequency, but two 4-decade resistance boxes, R_1 and R_2, allow each voltage to be varied. The outputs of the decade boxes, reversed in phase by means of an amplifier A, are added to the voltage drop across the resistance *r*, and the resultant voltage is displayed on a cathode-ray oscillograph. Under balance conditions the outputs of the two decade resistances are equal and opposite to the two components of the voltage drop across the resistance *r*, resolved along the directions of the two suppl[y] voltages, and the fundamental-frequency voltage disappears from the oscillograph screen. The admittance components of th[e] impedance *Z* can be obtained in terms of the resistance *r* an[d] the settings of the two decade resistances, and thus they do no[t] depend on any meter readings. The accuracy of measurement i[s] of the order of 0·1%, but when a large amount of noise i[s] present a tuned filter has to be included in the detecting circui[t] and the oscillograph can be replaced by an electronic wattmete[r].

Professor J. C. Prescott and Dr. A. K. El-Kharashi (*in reply*): As Mr. Thomas points out, the inductance measured at th[e] terminals of the circuit is a function of time because it is [a] function of a system of currents which are varying with time; for this reason we labelled the ordinate of Fig. 6 'appare[nt] inductance'.

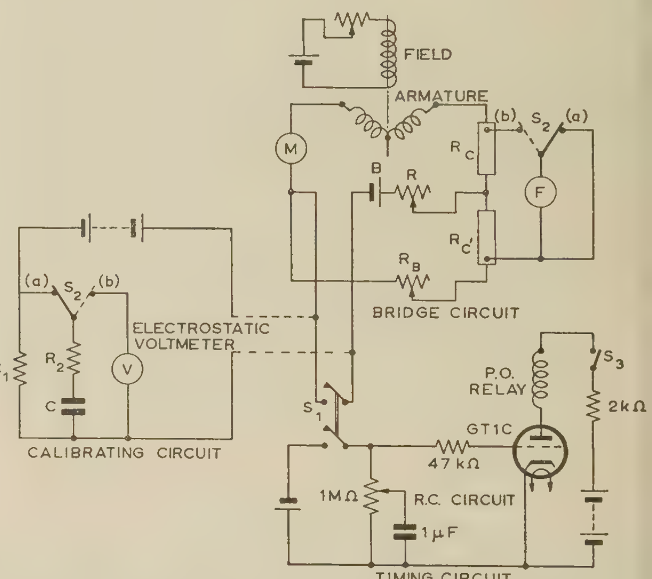

Fig. B.—Measurement of the apparent self-inductance of an armature as a function of time.

Fig. B shows the arrangements used to obtain the variation o[f] the armature flux linkages with time which is presented in Fig. 8. The time delay is defined by the discharge of a condenser whic[h] controls the grid voltage of a thyratron. A variable *RC* gri[d] circuit was used to give the required range of time delay. Th[e] Post Office relay in the anode circuit of the thyratron operate[s] switch S2.

The double-pole switch S_1 is closed first, and the curren[t] flowing through the armature and indicated by the ammeter M is adjusted to the required value. To obtain the armature flu[x] linkages under actual conditions, the current in the field circui[t] is also adjusted to any desired value. Since the relay coil is no[t] energized, S_2 is in position (*a*) short-circuiting the fluxmeter.

* PRESCOTT, J. C., and EL-KHARASHI, A. K.: Paper No. 2871 M, April, 1959 (see 106 A, p. 169).

† MORRIS, D.: 'A Phase-Calibrated Variable-Frequency Supply for the Testing of Servo Mechanisms', *Proceedings I.E.E.*, Paper No. 935 M, February, 1950 (97, Part II, p. 37).

The bridge circuit and the grid battery circuit are then opened simultaneously by switch S_1. When the grid voltage, now controlled by the *RC* network, reaches the striking value, the thyratron ignites and energizes the operating coil of the Post Office relay, which, in turn, changes over S_2 from position (*a*) to (*b*). The fluxmeter will therefore read the difference between the armature flux linkage at the instant when it is connected across the shunts R_C and $R_{C'}$ and its final steady value; this gives Φ_1 in Fig. 2. Switch S_3 is then opened to extinguish the thyratron discharge and the test is repeated for other time delays.

To measure the time delay of the *RC* circuit, the calibrating circuit on the left is inserted in place of the bridge. When S_1 is opened time elapses before the relay moves from position (*a*) to position (*b*) during which the condenser C discharges through the two resistors R_1 and R_2. The time delay is then given by $(R_1 + R_2)C \log_\varepsilon V_i/V_f$, where V_i and V_f are the initial and final voltages across the condenser.

Mr. Rudzinski describes a powerful method of measuring loss and apparent self-inductance as a function of frequency. From his results it would be possible to calculate the time-constant of a circuit at any frequency, but what is usually required in the case of large machines is the relationship between self-inductance and time. This is obtained directly by the method which we have described.

DISCUSSION ON 'ELECTRICAL MATERIALS AND COMPONENTS FOR AIRCRAFT POWER EQUIPMENT OPERATING AT HIGH TEMPERATURES'*

Mr. J. J. Lee (*communicated*): An appraisal of materials which I understand was made at the very latest some time in 1958 could not be entirely relevant for publication about a year later. As far as aluminium as a conductor material is concerned, the statements made appear to have been in error even at that time.

The references to aluminium and its alloys seem ambiguous. Does the author imply that the mechanical strengths of these materials fall below some arbitrary minimum or that the tensile strengths diminish more rapidly than copper? And what type of copper is the author considering? Copper conductors in insulated cables are generally annealed; it therefore seems relevant to compare copper in this condition with aluminium and aluminium alloy. Aluminium may have a tensile strength which is insufficient at the higher temperatures, but an aluminium-alloy conductor has adequate strength.

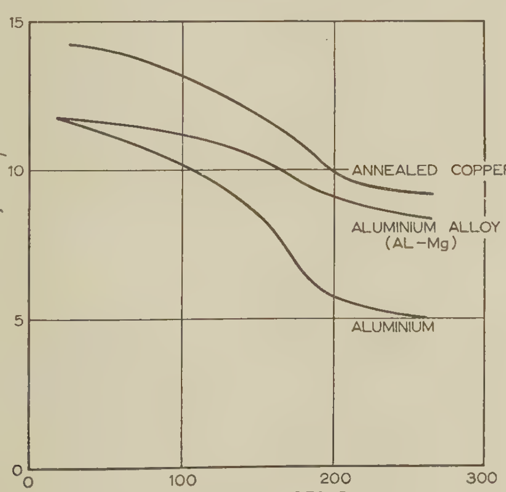

Fig. A.—Effect of elevated temperatures on tensile properties of conductor materials.

Fig. A shows the tensile properties of an aluminium alloy which is already being used as a conductor material, together with those of aluminium and annealed copper, but why does the author place paramount importance on tensile properties? Pure aluminium, although of relatively low strength at elevated temperatures, has been operating successfully.

For anodized-aluminium wire, reference is made to maximum operating temperatures of 300° C. Coils wound with this wire have operated continuously and satisfactorily at temperatures in the region of 500° C: porosity of the anodic film is not a drawback in many applications, particularly at very high temperatures, and with a suitable support this is one of the few materials that will withstand 400–500° C.

The author's views on aluminium cable terminations are surprising. These were originally produced in this country in accordance with M.O.S. Spec. R.D. Inst. (ELE) 147 issued in 1946. This was reasonably satisfactory for copper terminations,

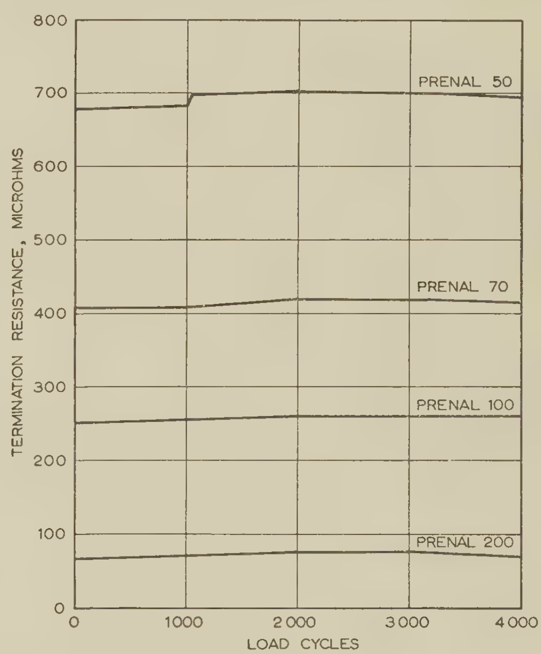

Fig. B.—Effect of sustained load and temperature cycling on joint resistance of cable terminations.

Cycle Details.—30 min heating, 30 min cooling. During heating period ambient temperature was 50° C plus that due to load current. Maximum temperature, 90° C. During cooling period ambient temperature fell to 30° C.

* McKenzie, D. B.: Paper No. 2912 U, August, 1959 (see 106 A, p. 321).

but the aluminium terminations have caused some trouble, due mainly to their bad design. I feel that it is on the performance of these terminations that the author bases his views. The Aluminium Development Association was first aware of their shortcomings in 1957 and immediately collaborated with several companies to determine the cause of failure of existing designs and to establish criteria for satisfactory designs.

Work has been conducted at a temperature of 90° C, and well-conceived designs incorporating a long hexagonal compression joint have proved to be satisfactory, as indicated in Fig. B.

I understand that since this programme was completed another company has been conducting some tests at 130° C which have also proved satisfactory. The Ministry of Supply has produced Technical Note EL154, which deals with tests on aluminium cable terminations and has made suggestions for good designs. Well-designed aluminium terminations are now commercially available.

The aluminium industry must view with the greatest concern the author's serious and damaging statements made in the light of conclusive evidence to the contrary.

Mr. D. B. McKenzie (*in reply*): In reply to Mr. Lee's comments on the use of aluminium on high-temperature aircraft equipment, his curve in Fig. A shows clearly that at 200° C the tensile strength of aluminium is 4 tons/in^2 less than that of annealed copper. That of aluminium-magnesium alloy at 200° C is about 1·5 tons/in^2 less than annealed copper, but annealed copper would not be used in any stressed electrical part of a component at high temperatures. As far as cables are concerned, they can be considered as unstressed parts, and, as such, aluminium will be satisfactory at, say, 200° C, but the terminations are the problem. It is agreed that the 'extended' aluminium lug suggested by the Aluminium Development Association is an improvement over the original type. For temperatures in the region of 100° C this appears to be the solution, and Fig. B supports this. There is, however, still too little experimental evidence and particularly flight experience to convince an aircraft engineer that aluminium cables and aluminium lugs at high temperatures will be satisfactory, but this information is being gathered together.

I agree that anodized-aluminium winding wires have been used very successfully in certain electrical applications, but on aircraft a wire whose insulation is porous is unacceptable unless it can be completely impregnated to prevent ingress of moisture. All aircraft spend much of their life on the ground under conditions of high humidity. Coils, consequently, become damp and this results in corrosion.

In reply to Mr. Lee's initial comment, the date of submission of the paper was 2nd December, 1958, and to the best of my knowledge it was up to date at that time.

DISCUSSION ON

'SUPPLY-VOLTAGE AND CURRENT VARIATIONS PRODUCED BY A 60-TON 3-PHASE ELECTRIC ARC FURNACE'*

Mr. G. J. Caplen (*communicated*): At the time this furnace transformer was manufactured it was approximately twice the size of the largest then built in this country, and the first with on-load tap-changing. In addition, the supply to the transformer was a temporary system pending extensions to the network, which have since taken place. All these factors had an influence on the design of the transformer. On smaller transformers it is usual to put the taps on the h.v. winding of the main transformer, but if that had been done in this case the following difficulties would have arisen:

(*a*) Equal number of turns per tap would not have given equal voltage per step on the l.v. winding.
(*b*) Severe over-voltages would have occurred in the h.v. winding.
(*c*) The operation of the tap-changer would have been more difficult.

The actual arrangement used has two 3-phase units mounted in the same tank, one being a fixed-ratio main furnace transformer, whilst the other supplies it with a variable voltage. The intermediate voltage of 25 kV maximum was chosen to give the most efficient operation of the tap-changers then available. Since 1954, we have manufactured seven furnace transformers to a rather more economic variation of this arrangement, which is shown in Fig. H. The sizes varied between 12.5 and 20 MVA, and the supply voltage was 33 kV in all cases except for the second furnace at this installation, where it is 66 kV.

* Robinson, B. C., and Winder, A. I.: Paper No. 2456 U, December, 1957 (see 105 A, pp. 305 and 578 and 107 A, p. 96).

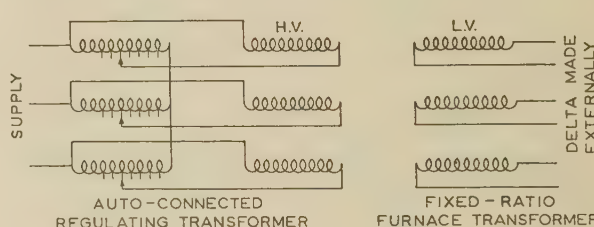

Fig. H

Both transformers are mounted in the same tank.

With regard to Mr. Care's suggestion of reactors controlling the voltage, this would be much dearer and less satisfactory in operation. I am not sure which winding Mr. Edwards wants to make inter-star, but it would certainly be much more expensive, and the mechanical strength of the transformer would be reduced. Several speakers have suggested obtaining supplies from 132 or 275 kV. I am doubtful of the economics of this at 15 MVA, but on larger sizes we have suggested combining the regulating unit with the Grid transformer. This effectively saves a transformer and should prove very economic where a single large unit is installed. It may be less economic, however, where a number of furnaces could be supplied from one Grid transformer.

Dr. B. C. Robinson and **Mr. A. I. Winder** (*in reply*): We should like to thank Mr. Caplen for his remarks on the design of furnace transformers and for amplifying points in our reply to the discussion at Birmingham.

DISCUSSION ON 'SELECTION OF RELAYING QUANTITIES FOR DIFFERENTIAL FEEDER PROTECTION'*

Mr. J. Rushton (*communicated*): The paper presents, for the first time, an analysis of the various combinations of phase-sequence quantities and serves to show that certain combinations are quite unsuitable for use as relaying quantities. The scope of the investigation covers single shunt-fault conditions on a simple power system, and in this respect Fig. 2 is rather misleading.

Essentially the paper calculates current at the point of fault for various shunt-fault conditions and expresses this current in terms of relaying quantities for various summation devices. The relationship between the total fault current and the fault currents at the relaying points, I_{FI} and I_{FII} (Fig. 2) is not immediately obvious, since these currents are identical only for the condition of single infeed with one circuit-breaker open. The ratio I_m/I_F must be identical in phase and magnitude for all three sequence currents for the method of analysis to be rigorous for the power system of Fig. 2. In general $I_{F1I}/I_{F1} = I_{F2I}/I_{F2} \neq I_{F0I}/I_{F0}$, so that, where the relaying quantity includes I_0, the method of analysis will contain an error. It might reasonably be assumed, however, that this unbalance will produce low output in one relay only, which can then be investigated as a single infeed condition after tripping has occurred at one end.

A completely rigorous analysis requires an investigation of the ratio $I_{mI}\angle\theta_I/I_{mII}\angle\theta_{II}$ to determine relay performance, particularly with more complicated types of fault on larger systems.

The authors' analysis indicates that the most satisfactory phase-sequence network arrangement for the conditions investigated is the $I_M = MI_2 - NI_1$ combination, and a further analysis of simultaneous shunt and series fault conditions on a double-circuit line might provide further valuable information.

The inherent limitation of summation-transformer output on resistance-earthed systems is shown clearly, although for the solidly earthed system, where R_0/X_1 is rarely greater than $0\cdot25$ even for high values of earth-electrode resistance, it would appear that the summation transformer provides a reliable relaying output in an essentially simple manner for all system fault conditions; and this is borne out by practical experience.

Mr. C. Adamson and **Dr. E. A. Talkhan** (*in reply*): It is time for the scope of investigation in the paper to be restricted to single-fault conditions; however, these comprise the great majority of faults on power systems. A number of more complicated fault conditions have also been worked out, but it is very difficult at the present time to ascribe any generality to this work. The most profitable line which is being pursued in the Power Systems Laboratory of the Manchester College of Science and Technology is to use a transformer-analogue network analyser for the investigation of a wider range of double and multiple fault conditions.

It appears from Mr. Rushton's comments that he may be under a misunderstanding, in part, of the method adopted in the paper. The currents considered in the paper are I_{FI} and I_{FII} at ends I and II, respectively; these currents are derived by normal analysis, assuming various shunt faults, but are quite independent of each other and are treated independently in the paper. In the ratio I_m/I_F, I_F is merely a convenient way of defining 1 p.u. fault current at the particular end under consideration; I_m/I_F is thus the p.u. relaying quantity at either end defined as the ratio of the output (from a sequence network of predetermined parameters) to the phase-fault current I_F at the *same* end. We disagree with Mr. Rushton that 'the ratio I_m/I_F must be identical in phase and magnitude for all three sequence currents for the method of analysis to be rigorous . . . '. In fact, as will be apparent from the foregoing, the quantity I_m is compounded of the positive-, negative- and zero-sequence currents at the end in question, in pre-chosen combinations, and may have any one or two of them absent, depending on the power system which is to be protected. In general, for protection purposes, it is sufficient to investigate the worst boundary conditions and deduce the limitations and precautions necessary under these conditions; this has been the aim of the work described in the paper and, for that matter, in a previous one.[6] With this in mind, a completely rigorous analysis of the complex ratio $I_{mI}\angle\theta_I/I_{mII}\angle\theta_{II}$, although of interest, is not really necessary. The difficulty is that there is no general system; it may be, however, that with a specific and more complicated power system it could be profitable to determine the complex ratio to assess P-relay performance under complicated fault conditions which are within the realms of engineering feasibility in the system being studied. Since there are an unlimited number of possible fault combinations, the work described in the paper is subject to endless elaboration, and some form of network analyser (the transformer-analogue network analyser is particularly convenient) set-up is essential.

We agree with the last paragraph of Mr. Rushton's constructive comments, but would point to one clear-cut case where the summation transformer is inadequate. This is the application where more than one output of different characteristics are required from the fault quantities of the power system. A good example may be taken from our earlier paper,[6] where a relaying quantity for modulation, $I_m = MI_2 - NI_1$, was required independently of another output, for starting, which had to have different characteristics.

* ADAMSON, C., and TALKHAN, E. A.: Paper No. 3137 M, February, 1960 (see 107 A, p. 37).

PAPERS AND MONOGRAPHS PUBLISHED INDIVIDUALLY

Summaries are given below of papers and monographs which have been published individually. The papers are free of charge; the price of the monographs is 2s. each (post free). Applications, quoting the serial numbers as well as the authors' names, and accompanied by a remittance where appropriate, should be addressed to the Secretary. For convenience, books of five vouchers, price 10s., can be supplied.

A General Method of Digital Network Analysis particularly suitable for use with Low-Speed Computers. Paper No. 3259 S.

M. N. John, B.Sc.

The concept of digital solutions of many of the network analysis problems associated with the design and operation of electrical power systems is now generally appreciated. There is, however, little indication of an increasing routine use of such methods in the United Kingdom, although many types of digital computer are now available. It is suggested that this is partly due to the fact that most of the existing programmes in this field employ techniques which have been prepared by skilled programmers for use with large high-speed computers, and digital analyses consequently tend to be restricted to the realm of special studies at computing centres.

The paper describes the network theory, programme details and application of a general method of digital network analysis particularly suitable for use with low-speed computers. The method has been proved using both high- and lower-speed computers and the results of sample load-flow and short-circuit studies are discussed.

A Survey of Street Lighting and its Future. Paper No. 3260 U.

W. R. Stevens, B.Sc., and H. M. Ferguson.

The accepted principles of street lighting are reviewed in conjunction with the requirements of the British Standard Code of Practice for Street Lighting. British and oversea techniques and standards are compared, and some important recent experimental work and installations are described. These factors are used to assess the desirable trend of street lighting in the future.

Brushless Variable-Speed Induction Motors using Phase-Shift Control. Paper No. 3262 U.

Prof. F. C. Williams, O.B.E., D.Sc., D.Phil., F.R.S., E. R. Laithwaite, M.Sc., Ph.D., J. F. Eastham, M.Sc., and W. Farrer, B.Sc.

The paper describes a method of 'pole-stretching' for induction machines in which part of the stator windings are fed directly from the mains supply and part from phase-shifting transformers. Variation of the angle of phase-shifting enables continuous speed control to be effected. An experimental machine is described, the test results from which demonstrate that speed control with constant efficiency can be obtained over a speed range of 1·6 : 1. The limitations on the range of such machines imposed by the necessary condition that the stator be discontinuous are discussed, and a method of extending the speed range is then described. Machines of this type may be designed to run with a number of discrete synchronous speeds, in which case no phase-shifting transformers are necessary and speed change is effected by external switches only. The historical link between this type of machine and the spherical motor is outlined.

Investigation of an Electrical Non-Destructive Method of Measuring the Depth of Surface Hardness in Flame-Hardened Steels. Monograph No. 372 M.

J. A. Betts, B.Sc., Ph.D., and J. P. Newsome, M.Sc.

At the present time there exist no established, non-destructive methods for the measurement of depth of hardness in surface-hardened steels which are independent of the effects of chemical composition and quench procedure. Electrical non-destructive methods are dependent upon changes in the electrical and magnetic properties of steel which occur when it is hardened.

The electrical method investigated by the authors was an a.c. one based upon the measurement of the complex impedance of a search coil magnetically coupled to the test surface. Distinctly favourable results were obtained, and the paper is concerned with the theoretical and practical aspects of the procedure.

A New Form of the Tensor Equations of Electrical Machines. Monograph No. 378 S.

G. S. Brosan, Ph.D., B.Sc.(Eng.).

Previous tensor methods applied to electrical engineering systems have been based on Lagrange's equations, which are not always suitable for non-holonomic systems. As a consequence, the defining equations lose much of their simplicity and the component terms in the equations may no longer be tensors.

The paper presents a new set of equations based on the principle of least curvature. These equations are suitable for both holonomic and non-holonomic systems, and their application is shown by examples.

Electric and Magnetic Images. Monograph No. 379.

P. Hammond, M.A.

The method of images as applied to electrostatic, magnetostatic and electromagnetic fields is investigated. By considering the uniqueness of the field it is shown within what limits the method can safely be used, and rules are given for its use. The application of the method is illustrated by a discussion of the electric field near a cylindrical cathode and the magnetic field near the end-windings of electrical machines.

Equivalent Circuit and Evaluation of Eddy-Current Loss in Solid Cores subjected to Alternating and Rotating Magnetic Fields. Monograph No. 385 U.

N. Kesavamurthy, M.A., B.E., M.Sc., and P. K. Rajagopalan, B.E., M.S., Ph.D.

In the study of eddy-current distribution in solid cores subjected to a pulsating or rotating magnetic field a general two-dimensional current distribution exists and is created by a magnetizing winding that restricts the current flow through it in only one direction. Under such situations, the method of evaluation of the equivalent impedance of the system as a whole is difficult to visualize and has not so far been attempted in the literature. Such an evaluation calls for a concept of the power factor of the magnetizing winding. This is the special feature of the paper.

The paper further examines the usefulness of this concept for the evaluation of eddy-current loss in solid cores and thus offers an alternative method.

To give greater concreteness to several of the formulae deduced in the paper, these are applied to compute the performance characteristics of a solid-rotor induction machine and certain conclusions are drawn.

An Analytical Method for Predicting the Performance of Semi-Enclosed Fuses. Monograph No. 387 S.

Colin Adamson, M.Sc.(Eng.), and M. Viseshakul, M.Sc.Tech.

An analysis has been made and a design procedure established for fuses other than those using round wire and with, or without, a restricted cross-section for part of the fuse length. Fuses of restricted section are, however, the major interest since they may be used to give different current/time characteristics by varying their dimensions. These different characteristics may be predicted accurately by the method indicated in the paper. In all the cases of semi-enclosed tin strip fuses investigated in this way very close agreement between predicted and experimental results has been obtained.

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Part A. POWER ENGINEERING, JUNE 1960

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Example.—SMITH, J.: "Overhead Transmission Systems," *Proceedings I.E.E.*, Paper No. 4001 S, December, 1954 (102 A, p. 1234).

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